Problem Solving Using Congruence

Problem 1: When two different numbers are divided by 11, remainders of 7 and 9, respectively, are left. What is the remainder when the sum of these two numbers is divided by 11?

Problem 2: When two different numbers a and b are divided by 7, remainders of 2 and 3, respectively, are left. What is the remainder when $b - a$ is divided by 7?

Problem 3: When a is divided by 5, the remainder is 1. When b is divided by 5, the remainder is 4. If $3a > b$, what is the remainder when $3a - b$ is divided by 5?

Problem 4: Find the smallest number that gives a remainder of 1 when divided by 4, a remainder of 2 when divided by 5, and a remainder of 3 when divided by 6.

Problem 5: Mary thought of an integer greater than 10, that when divided by 2, 3, 4, 5, or 6 has a remainder one less than the divisor. When divided by 7 the remainder is one. Find the smallest such integer.

Problem 6: When a is divided by 7, the remainder is 3. When b is divided by 7, the remainder is 5. If $a^2 > 4b$, what is the remainder when $a^2 - 4b$ is divided by 7?

Problem 7: (AMC) For each integer $N > 1$, there is a mathematical system in which two or more integers are defined to be congruent if they leave the same non-negative remainder when divided by N. If 69, 90, and 125 are congruent in one such system, then in that same system, 81 is congruent to
(A) 3 (B) 4 (C) 5 (D) 7 (E) 8

Problem 8: (AMC) The greatest integer that will divide 13,511, 13,903 and 14,589 and leave the same remainder is
(A) 28 (B) 49 (C) 98 (D) an odd multiple of 7 greater than 49
(E) an even multiple of 7 greater than 98

Problem 9: (AMC) If r is the remainder when each of the numbers 1059, 1417 and 2312 is divided by d, where d is integer greater than one, then $d - r$ equals
(A) 1 (B) 15 (C) 179 (D) $d - 15$ (E) $d - 1$

Problem 10. (Mathcounts Competitions) In how many ways can a debt of $69 be paid exactly using only 5-dollar and 2-dollar bills?

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Problem 11 (1998 Mathcounts National Team #3) Exactly one ordered pair of positive integers (x, y) satisfies the equation $37x + 73y = 2016$. What is the sum of $x + y$?

Problem 12. (Mathcounts 1988 National Team#10) A developer has 87 acres and he would like to divide it into smaller lots. Some should be 2 acres, some should be 3 acres, and some should be 5 acres. If the developer must have exactly 25 lots (allowing no fractional parts of lots), and at least one lot of each type, how many different ways can he divide up the 87 acres?

Problem 13. Let $2x + 3y = 72$. How many ordered pairs (x, y), where x and y are whole numbers, will satisfy the equation? (2002 Mathcounts National).

Problem 14. (AMC) The number of solution-pairs in positive integers of the equation $3x + 5y = 501$ is:
(A) 33 (B) 34 (C) 35 (D) 100 (E) none of these.

Problem 15: Katrina and Abbie start a game with one pile of 40 pennies. They take turns. On each turn, a player must take 1, 2, 3, 4 or 5 pennies from the pile. The player who takes the last penny from the pile of 40 pennies wins the game. If Abbie plays first, what number of pennies must she take from the pile on her first turn in order to guarantee that she can win the game?

Problem 16. A proposal will make years that end in double zeroes a leap year only if the year leaves a remainder of 200 or 600 when divided by 900. Under this proposal, how many leap years will there be that end in double zeroes between 1996 and 4096? (Mathcounts handbooks)

Problem 17. (1992 Mathcounts National Team) What are the last two digits of 2^{222}?

Problem 18. (1983 AIME) Let a_n equal $6^n + 8^n$. Determine the remainder upon dividing a_{83} by 49.

Problem 19. How many positive integers from1 to 2019 are there such that $2^n - 1$ is divisible by 7?

Problem 20: Take out n distinct numbers from $\{1, 2, 3, \ldots, 2019\}$ such that the sum of any three of these n numbers is divisible by 18. What is the largest possible value of n?