Math Competition Books Series

The Mass Points Method

Yongcheng Chen, Ph.D.

http://www.mymathcounts.com/index.php

ACKNOWLEDGEMENTS

We would like to thank Mr. Thomas Rike, the author of "A Decade of the Berkeley Math Circle – the American Experience". This book was inspired by his article: "Mass Point Geometry".

This book introduces a powerful problem solving technique − the mass points method. The book can be used by students preparing for math competitions such as Mathcounts, AMC 10/12/AIME.

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Please contact mymathcounts@gmail.com for suggestions, corrections, or clarifications.

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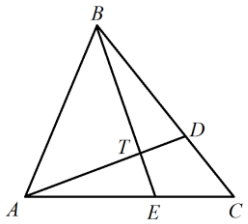
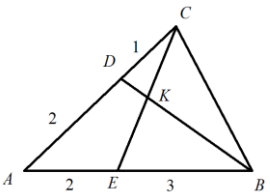
2. Direct Application of Mass Points

Example 5. (2004 AMC 10B Problem 20) In ΔABC, points D and E lie on BC and AC, respectively. If AD and BE intersect at T so that $\frac{AT}{DT}=3$ and $\frac{BT}{ET}=4$, what is $\frac{CD}{BD}$?

(A) $\frac{1}{8}$ (B) $\frac{2}{9}$ (C) $\frac{3}{10}$ (D) $\frac{4}{11}$ (E) $\frac{5}{12}$

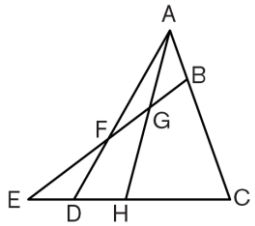


Example 7. (2011 National Team 9) In triangle ABC, point D is on segment AC with $AD : DC = 2 : 1$, and point E is on segment AB with $AE : EB = 2 : 3$. Segments EC and DB intersect at point K. What is $DK : KB$? Express your answer as a common fraction.

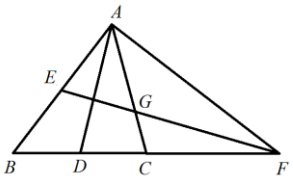


3. Removing Line Segments

Example 4. (Mathcounts) For integers x, y and z, if $AB:BC = 1:4$, $AG:GH = 3:5$ and $AF:DF = 5:4$, then $CH:DH:DE = x : y : z$. What is the value of $x + y + z$?



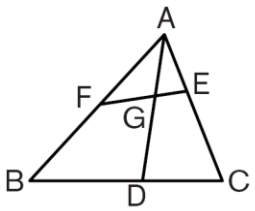
Example 5. As shown in the figure, AD is the angle bisector of $\angle A$ of triangle ABC. The perpendicular bisector of AD meets AB at E, AC at G, and the extension of BC at F. Find $\frac{CF}{BF}$ if $\frac{AC}{AB}=\frac{3}{4}$.



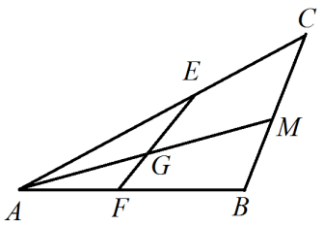
Mass Points Method Sample Problems

4. Splitting Mass Points

Example 3. (Mathcounts) Triangle ABC, shown here, has cevian AD and transversal EF intersecting at G, with $AE : CE = 1 : 2$, $AF : BF = 5 : 4$ and $BD : CD = 3 : 2$. What is the ratio of AG to DG? Express your answer as a common fraction.

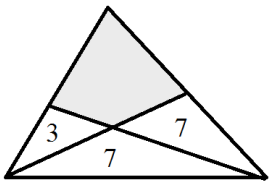


Example 4. (AMC) In triangle ABC shown in the adjoining figure, M is the midpoint of side BC, $AB = 12$ and $AC = 16$. Points E and F are taken on AC and AB, respectively, and lines EF and AM intersect at G. If $AE = 2AF$ then EG/GF equals
(A) 3/2 (B) 4/3 (C) 5/4 (D) 6/5
(E) not enough information given to solve the problem

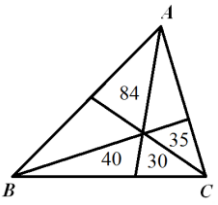


5. Length and Area Problems

Example 3. (2006 AMC 10 B Problem 23) A triangle is partitioned into three triangles and a quadrilateral by drawing two lines from vertices to their opposite sides. The areas of the three triangles are 3, 7, and 7, as shown. What is the area of the shaded quadrilateral?
(A) 15 (B) 17 (C) 35/2 (D) 18 (E) 55/3

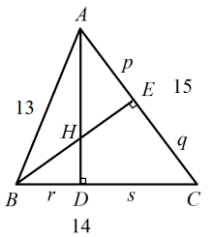


Example 7. (AIME) As shown in the figure on the right, ΔABC is divided into six smaller triangles by lines drawn from the vertices through a common interior point. The areas of four of these triangles are as indicated. Find the area of ΔABC.



Mass Points Method Sample Problems

Example 9. (AMC) The sides of a triangle are of lengths 13, 14, and 15. The altitudes of the triangle meet at point H. If AD is the altitude to side of length 14, the ratio $HD : HA$ is:
(A) 3: 11 (B) 5: 11 (C) 1:2 (D) 2:3 (E) 25:33



Example 16. (2002 AIME II) In triangle ABC, point D is on BC with $CD = 2$ and $DB = 5$, point E is on AC with $CE = 1$ and $EA = 3$, $AB = 8$, and AD and BE intersect at P. Points Q and R lie on AB so that PQ is parallel to CA and PR is parallel to CB. It is given that the ratio of the area of triangle PQR to the area of triangle ABC is m/n, where m and n are relatively prime positive integers. Find $m + n$.

Problem 4. (2013 Mathcounts National Sprint 28) In right triangle ABC, shown here, $AC = 5$ units and $BC = 12$ units. Points D and E lie on AB and BC , respectively, so that CD is perpendicular to AB and E is the midpoint of BC . Segments AE and CD intersect at point F. What is the ratio of AF to FE? Express your answer as a common fraction.