Example 1. Evaluate $\frac{ab^2(c^2-5)}{abc}$ when $a = \frac{1}{3}$, $b = -\frac{2}{7}$, and $c = \frac{4}{5}$. Express your answer to the nearest hundredths.

Solution: 1.56.
The expression $\frac{ab^2(c^2-5)}{abc}$ can be evaluated "when" $a = \frac{1}{3}$, $b = -\frac{2}{7}$, and $c = \frac{4}{5}$ by entering the following on the command line:

x×y^2(z^2-5)÷(x×y×z) | *x* = 1÷3 CATALOG ►and y = (-)2÷7 ENTER

2ND (-) | z = 4÷5.0 ENTER
The result is 1.55714.
The answer is 1.56.

Example 2. (2010 Mathcounts Chapter Target) If $a + b + c + d = 11$, $2a + 3c = 19$, $b + 4d = 22$, $4a + d = 14$ and $5b + 3c = 5$, what is the value of d?

Solution: 6.
Method 1 (official solution):
$a + b + c + d = 11$
$2a + 3c = 19$
$b + 4d = 22$
$4a + d = 14$
$5b + 3c = 5$
So what is d? Let's start by adding together the last 4 equations.
$2a + 3c = 19$
$+ b + 4d = 22$
$+4a + d = 14$
$+ 5b + 3c = 5$
$6a + 6b + 6c + 5d = 60$
Notice that all of the variables have a coefficient of 6 except d. That means we can multiply the first equation by 6 and subtract from it the sum of the other 4 equations and will be left with d.
$6a + 6b + 6c + 6d = 66$
$6a + 6b + 6c + 5d = 60$
$d = 6$.

Method 2:
press F2 (Algebra) and select choice 1: solve(.
Input:
solve(x+y+z+t=11 CATALOG ► and 2x+3z=19 CATALOG ► and y+4t=22 CATALOG ► and 4x+t=14 CATALOG ► and 5y+3z=5, 2ND (x,y,z,t 2ND)) ENTER

The screen will show: t = 6 and x = 2 and y = – 29 and z = 5.
The answer is $d = 6$.

Example 3. (2009 Mathcounts State Team) The mean of a set of five positive integers is 1.5 times the median. If three of the integers in the set are 24, 52 and 86, and one of them is the median, what is the sum of all distinct possible sums of the other two integers?

Solution: 729.

Method 1(official solution):

The mean of a set of 5 positive integers is 1.5 times the median. Three of the integers in the set are 24, 52 and 86. One of these values is the median. We are asked to determine the sum of all distinct possible sums of the other two integers. Let x and y be the other 2 values. If 24 is the median, we have

$$x + y + 24 + 52 + 86 = x + y + 162$$

$$\frac{x+y+162}{5} = 24 \times 1.5 = 36$$

$$x + y + 162 = 180$$

$$x + y = 18$$

Now suppose that 52 is the median.

$$\frac{x+y+162}{5} = 52 \times 1.5 = 78$$

$$x + y + 162 = 390$$

$$x + y = 228$$

Finally, suppose that 86 is the median.

$$\frac{x+y+162}{5} = 86 \times 1.5 = 129$$

$$x + y + 162 = 645$$

$$x + y = 483$$

18 + 228 + 483 = 729

Method 2:

Let five numbers be a, b, 24, 52, 86. Let $y = a + b$. Let x be the median.

Since $\frac{y+24+52+86}{5} = x \times 1.5 \Rightarrow \quad y + 24 + 52 + 86 = x \times 1.5 \times 5$

$\Rightarrow \quad y = x \times 1.5 \times 5 - 24 - 52 - 86$

Entering the following on the command line:

F4 1: Define ENTER

ALPHA I (x) = $x \times 1.5 \times 5 - 24 - 52 - 86$ ENTER

DONE

ALPHA I (24) + ALPHA I (52) + ALPHA I (86) ENTER

The result is 729.

Example 4. (2009 Mathcounts Handbook) At what point does the graph of $3x + 4y = 15$ intersect the graph of $x^2 + y^2 = 9$? Express any non-integer coordinate as a common fraction.

Solution: (9/5, 12/5).
Method 1 (official solution):
The graph of the equation $3x + 4y = 15$ is a line, and the graph of the equation $x^2 + y^2 = 9$ is a circle. The line might not cross at all, it might touch at just one tangent point, or it might cross the circle twice. If we solve the linear equation for y, we get $y = -(3/4)x + 15/4$. If we solve the circle equation for y, we get $y = \sqrt{9 - x^2}$. If these two graphs intersect, then their y values are equal at that point, so we can set these two equations equal and solve for x. We will try the positive value for y first. The algebra is shown below:

$\sqrt{9 - x^2} = -\frac{3}{4}x + \frac{15}{4}$

$4\sqrt{9 - x^2} = -3x + 15$

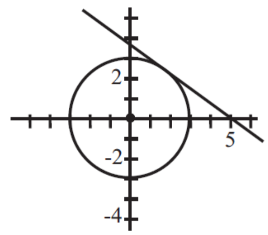
$\left(4\sqrt{9 - x^2}\right)^2 = (-3x + 15)^2$

$16(9 - x^2) = 9x^2 - 90x + 225$

$144 - 16x^2 = 9x^2 - 90x + 225$

$0 = 25x^2 - 90x + 81$

$0 = (5x - 9)(5x - 9)$



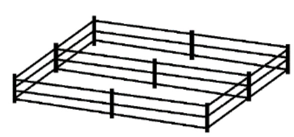
The fact that we got the same solution twice, namely $x = 9/5$, means that the line is tangent to the circle. Substituting $x = 9/5$ into the linear equation, we get $y = -(3/4)(9/5) + 15/4 = -27/20 + 75/20 = 48/20 = 12/5$. The single point of intersection is (9/5, 12/5).

Method 2:
press F2 (Algebra) and select choice 1: solve(.

Input:
solve(3x+4y=15 CATALOG ► and x^2+y^2=9, 2ND (x,y 2ND))   ENTER

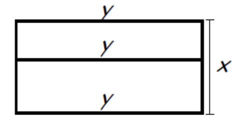
The screen will show: x = 9/5 and y = 12/5.
The single point of intersection is (9/5, 12/5).

Example 5. (2005 Mathcounts Handbook Workout) Keli and Mario are planning to plant rectangular gardens of the same length, side by side with fencing all around and dividing the two plots. The total amount of fencing is 100 feet. If the total area of the two plots is 336 square feet and the dimensions are integers, what is the length of the fence that divides the two plots?



Solution: 24.
Method 1(official solution):

First, we can add some variables to our picture. We don't know that the fencing between the two gardens splits the entire area in half, but we know $xy = 336$ and $3y + 2x = 100$. We now need to solve for y. We have two equations and two unknowns, so it is possible to solve at this point.



However, even after getting these merged into one equation with one variable, it is going to be challenging to solve. Starting with $xy = 336$, we can divide both sides by y to see that $x = \frac{336}{y}$.
Now we have $3y + 2(\frac{336}{y}) = 100$; $3y^2 + 2(336) = 100y$; $3y^2 - 100y + 672 = 0$;
$(3y - 28)(y - 24) = 0$. From here, we see that $y = \frac{28}{3}$ or 24. Since we were told the dimensions were integers, we have $y = 24$.

Alternatively, we could use our calculator with some Guess, Check & Revise and perhaps get to the answer quicker. Since $3y + 2x = 100$, we know that both x and y must be less than 50. (Since they.re both positive integers, 50 is a good estimate cap since $3(0) + 2(50) = 100$.) We also know that $xy = 336$. We don't need to check factor pairs of 336 with extremely large factors like 336 & 1 or 168 & 2 because of the cap we.ve already established. Our factor pair is going to have to consist of numbers that are both closer to the square root of 336 (which is approximately 18.3) than the extreme values. The prime factorization of 336 is $2^4 \times 3 \times 7$. We can.t form the factors 18, 19 or 20 from these prime factors, but let's look at a pair of factors involving 21. The factor pair is 21 & 16. This pair must satisfy $3(21) + 2(16) = 100$ or $3(16) + 2(21) = 100$. It fails, but both are close. We can't form 22 or 23 from the factorization of 336, but let's examine 24. The factor pair would be 24 & 14. Upon testing $3(24) + 2(14)$, we see the result is 100, and so $y = 24$.

We also could use the TABLE feature of a graphing calculator to solve this problem. We will use the relationships $y = \frac{336}{x}$ and $3y + 2x = 100$. First, we will enter the following equations in the .Y=. field: $Y1 = \frac{336}{x}$ and $Y2 = 3(\frac{336}{x}) + 2x$. When we go to the TABLE feature, the first column will show a value of x, the Y1 column will show the corresponding value of y (notice the product of every pair of values in these first two columns is 336), and the value in the Y2 column is the corresponding value of $3y + 2x$. If we scroll down until we have 100 in the Y2 column, we can see that this happens only when $x = 14$ and $y = 24$.

Method 2:
press F2 (Algebra) and select choice 1: solve(.

Input:
solve(x×y=336 CATALOG ► and 3y + 2x = 100, 2ND (x,y 2ND)) ENTER

The screen will show: x = 14 and y = 24 or x = 36 and y = 28/3.
Since the dimensions are integers, the length of the fence must be 24.

http://samsmathclub.com/Forum/index.php?board=207.0