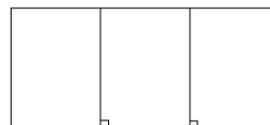


1. (Mathcounts) Two natural numbers have a sum of 23. What is their largest possible product?

2. (Mathcounts) The sum of two positive integers is eleven. What is the smallest possible sum of their reciprocals? Express your answer as a common fraction.

3. (Mathcounts) If a and b are numbers such that $a + b = 18$, what is the smallest value of $a^2 + b^2$?

4. (1994 Mathcounts National Target problem 8) A farmer has 60 meters of fence with which to build a rectangular animal run (as shown) for his cows, horses, and pigs. She wants each type of animal to have the same area. What is the largest number of square meters which can be enclosed? Express your answer as a decimal.



5. Let $a > 5$, find the smallest value of $a + \frac{4}{a-5}$.

6. A farmer wishes to use 4 miles of fencing to enclose a rectangular region of greatest area. However, one side runs along a river, so only three sides must be fenced in. What is the largest possible area of the rectangular region that can be enclosed?

7. (AMC) For $x \geq 0$, what is the smallest value of $\frac{4x^2 + 8x + 13}{6(x+1)}$?

- (A) 1 (B) 2 (C) $\frac{25}{12}$ (D) $\frac{13}{6}$ (E) $\frac{34}{6}$

8. A coffee filter is to be made from a circle of radius 1 with a sector with the central angle of x cut from it such that V , the volume of coffee held in the filter is maximized. What is the volume V ?

9. (2008 AIME) A block of cheese in the shape of a rectangular solid measures 10 cm by 13 cm by 14 cm. Ten slices are cut from the cheese. Each slice has a width of 1 cm and is cut parallel to one face of the cheese. The individual slices are not necessarily parallel to each other. What is the maximum possible volume in cubic cm of the remaining block of cheese after ten slices have been cut off?

10. (USAMTS) Find the ordered pair of real numbers (x, y) that satisfies the equation below, and demonstrate that it is unique.

$$\frac{36}{\sqrt{x}} + \frac{9}{\sqrt{y}} = 42 - 9\sqrt{x} - \sqrt{y}.$$

11. (AMC) The number of real solutions (x, y, z, w) of the simultaneous

equations $2y = x + \frac{17}{x}$, $2z = y + \frac{17}{y}$, $2w = z + \frac{17}{z}$, $2x = w + \frac{17}{w}$ is

- (A) 1 (B) 2 (C) 4 (D) 8 (E) 16