American Mathematics Competitions

Practice 10

AMC 10

(American Mathematics Contest 10)

INSTRUCTIONS

1. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.

2. You will have 75 minutes to complete the test.

3. No aids are permitted other than scratch paper, graph paper, rulers, and erasers. No problems on the test will require the use of a calculator.

4. Figures are not necessarily drawn to scale.

5. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.

1. One can holds 15 ounces of soda. What is the minimum number of cans needed to provide three gallons (1 gallon = 128 ounces) of soda?

(A) 27 (B) 26 (C) 25 (D) 20 (E) 18

2. The sums of three whole numbers taken in pairs are 17, 18, and 25. What is the middle number?

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

3. Simplify as a common fraction: $\dfrac{1}{1+\dfrac{1}{1+\dfrac{2}{3}}}$.

(A) 1/8 (B) 2/8 (C) 3/8 (D) 5/8 (E) 7/8

4. Thirty percent less than 70 is two-fifth more than what number?

(A) 26 (B) 30 (C) 32 (D) 35 (E) 48

4. D.

5. Kathy has two younger twin brothers. The product of their three ages is 1024. What is the smallest possible sum of their three ages?

(A) 32 (B) 24 (C) 22 (D) 18 (E) 16

6. In a class of 42 students, 18 students are in the Math Club, 5 students are in both the Math Club and the Science Club, and 14 are in neither. How many students are in the Science Club?

(A) 5 (B) 15 (C) 20 (D) 35 (E) 30

7. The number of centimeters in the length, width and height of a rectangular carton are consecutive integers. Find the smallest 4-digit number that could represent the number of cubic centimeters in the volume)

(A) 1001 (B) 1320 (C) 1331 (D) 1025 (E) 1216

8. A majority of the 40 students in Ms. Li's class bought pencils at the school bookstore. Each of these students bought the same number of pencils, and this number was greater than 1. The cost of a pencil in cents was greater than the number of pencils each student bought, and the total cost of all the pencils was \$20.15. How many students in the class bought the pencils?

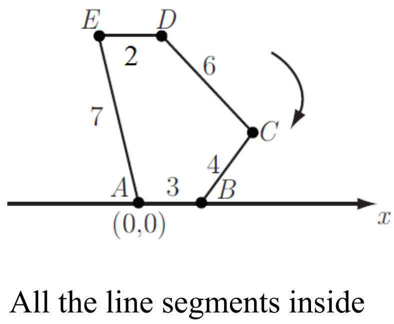
(A) 35 (B) 11 (C) 33 (D) 13 (E) 31

9. Which of the following is equal to Simplify $\sqrt{6-\sqrt{11}} + \sqrt{6+\sqrt{11}}$.

(A) $\sqrt{22}$ (B) 12 (C) $12+\sqrt{22}$ (D) $6+\sqrt{22}$ (E) $2\sqrt{3}$

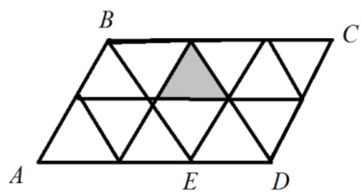
10. As shown below, convex pentagon *ABCDE* has sides *AB* = 3, *BC* = 4, *CD* = 6, *DE* = 2, and *EA* = 7. The pentagon is originally positioned in the plane with vertex *A* at the origin and vertex *B* on the positive *x*-axis. The pentagon is then rolled clockwise to the right along the *x*-axis. Which side will touch the point *x* = 2015 and be completely on the *x*-axis?

(A) *AB* (B) *BC* (C) *CD* (D) *DE* (E) *EA*



11. *ABCD* is a parallelogram. All the line segments inside the figure either parallel to *AD, AD,* or *BE*. How many parallelograms are there that contain the shaded triangle?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 12



12. A 5 × 5 × 5 wooden cube is painted on five of its faces and is then cut into 125 unit cubes. One unit cube is randomly selected and rolled. What is the probability that the face showing is painted? Express your answer as a fraction.

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{5}$ (D) $\frac{2}{7}$ (E) $\frac{28}{125}$

13. The lengths of the parallel bases of a trapezoid are 14 cm and 7 cm. One of the legs has length 8 cm. How many integer values are possible for the length of the other leg?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 13

14. It takes Amy 4 hours to paint a house, it takes Bill 6 hours, and it takes Chandra 8 hours. Amy starts to paint the house for one hour, then Bill continues the job for another hour, and Chandra follows Bill and works for one hour. If the pattern continues until the job is completed, how many hours does Chandra paint the house?

(A) $\frac{4}{3}$ (B) $\frac{1}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) $\frac{2}{3}$

15. Alex can jog 120 meters in 1 minute, Bob can jog 80 meters in 1 minute, and Charlie can jog 70 meters in 1 minute. The circular path has a circumference of 1000 meters. They start running together at 10:00 a.m. at point A. At what time will they first all be together again at point A?

(A) 10: 40 a.m. (B) 10:50 a.m. (C) 11:20 a.m. (D) 11:40 a.m. (E) 11:55 a.m.

16. We are choosing a committee of 6 animals from 3 cats, 4 dogs, and 5 pigs. If Alex Cat, Bob Dog and Charles Pig do not like each other and they will not work in the same group, how many compatible committees are there?
(A) 84 (B) 220 (C) 304 (D) 378 (E) 462

17. What is the remainder when $3^1 + 3^2 + \cdots + 3^{2015}$ is divided by 8?

(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

18. The capacity of a car's radiator is nine liters. The mixture of antifreeze and water is 40% antifreeze. The temperature is predicted to drop rapidly requiring the mixture to be 70% antifreeze. How much of the mixture in the radiator must be drawn off and replaced with pure antifreeze?

A. 3.5 liters B. 4.5 liters C. 5.0 liters D. 6.0 liters E. none of these

19. The sum of a four-digit positive integer and its digits is exactly 2015. Find the sum of all possible values of the four-digit positive integer.

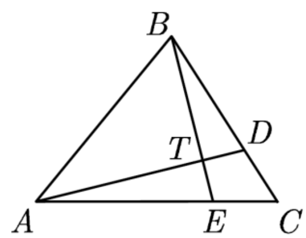
(A) 2015 (B) 2012 (C) 4004 (D) 4008 (E) 4030

20. Pipe A will fill a tank in 6 hours. Pipe B will fill the same tank in 4 hours. Pipe C will fill the tank in the same number of hours that it will take Pipes A and B working together to fill the tank. What fraction of the tank will be filled if all three of the pipes work together for one hour?

(A) $\frac{4}{3}$ (B) $\frac{5}{3}$ (C) $\frac{5}{6}$ (D) 1 (E) $\frac{2}{3}$

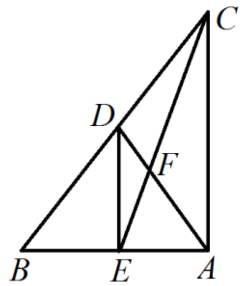
21. In ΔABC points D and E lie on BC and AC, respectively. If AD and BE intersect at T so that $AT/DT = 5$ and $BT/ET = 7$, what is CD/BD?

(A) $\frac{3}{17}$ (B) $\frac{5}{7}$ (C) $\frac{4}{17}$ (D) $\frac{4}{7}$ (E) $\frac{1}{7}$



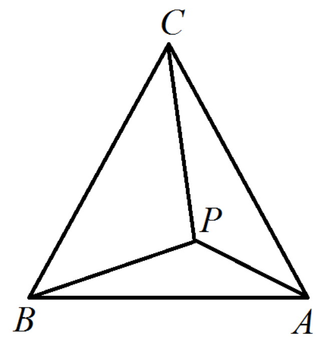
22. The area of right triangle ABC is 360. $\angle BAC = 90°$. AD is the median on BC. $DE \perp AB$. AD an CE meet at F. Find the area of triangle AEF.

(A) 60 (B) 70 (C) 80 (D) 90 (E) 100



23. P is a point inside the equilateral triangle ABC. $PA = 2$, $PB = 2\sqrt{3}$, and $PC = 4$. Find the area of triangle ABC.

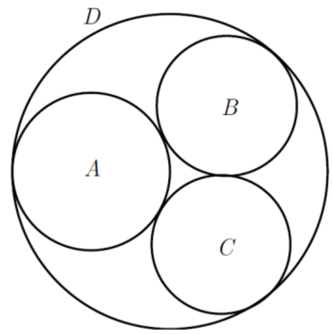
(A) $7\sqrt{3}$ (B) $4\sqrt{3}$ (C) $7\sqrt{38}$ (D) $8\sqrt{38}$
(E) $7\sqrt{2}$



24. A special deck of cards contains cards numbered 1 through 5 for each of five suits. Each of the 25 cards has a club, diamond, heart or spade on one side and the number 1, 2, 3, 4 or 5 on the other side. After a dealer mixed up the cards, three were selected at random. What is the probability that of these three randomly selected cards, displayed here, one of the cards showing the number 3 has a spade printed on the other side? Express your answer as a common fraction.

(A) $\frac{4}{11}$ (B) $\frac{4}{9}$ (C) $\frac{25}{54}$ (D) $\frac{5}{11}$ (E) $\frac{5}{9}$

25. Circles *A*, *B*, and *C* are externally tangent to each other and internally tangent to circle *D*. Circles *B* and *C* are congruent with radius 8. Circle *A* passes through the center of *D*. What is the radius of circle *A*?



(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

ANSWER KEYS

1. B.
2. B.
3. D.
4. D.
5. A.
6. B.
7. B.
8. E.
9. A.
10. C.
11. E.
12. A.
13. E.
14. A.
15. D.
16. E.
17. A.
18. B.
19. C.
20. C.
21. A.
22. A.
23. A.
24. B.
25. D.

SOLUTIONS:

1. B.

Because 3 × 128 / 13 = 25.6, there must be 26 cans.

2. B.

Let three numbers be a, b, and c.

$a + b = 17$ (1)

$b + c = 25$ (2)

$c + a = 18$ (3)

(1) + (2) + (3): $2(a + b + c) = 60 \quad \Rightarrow \quad a + b + c = 30$ (4)

(4) – (3): $b = 12$.

3. D.

$$\frac{1}{1+\frac{1}{1+\frac{2}{3}}} = \frac{1}{1+\frac{1}{\frac{5}{3}}} = \frac{1}{1+\frac{3}{5}} = \frac{1}{\frac{8}{5}} = \frac{5}{8}.$$

4. D.

Thirty percent less than 70 is $70 - \frac{30}{100} \times 70 = \frac{2}{5}x + x \quad \Rightarrow \frac{7}{5}x = 49 \quad \Rightarrow x = 35$

5. A.

The age of each person is a factor of $1024 = 2^{10}$. Since we want the smallest sum, the three numbers should be as close as possible.

$2^{10} = 2^3 \times 2^3 \times 2^4$.

The twins could be 8, and 8. Kathy could 16.

The smallest sum of their ages is 8 + 8 + 16 = 32.

6. B.

There are 42 – 14 = 28 students participated in the two clubs. Let S be the number of students in the Science Club.

By the Two Events Union Formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, we have 28 = 18 + S – 5. So $S = 15$.

7. B.

Let the smallest value of the length, width and height be $a - 1$. Since the numbers are consecutive integers, then the other two dimensions are a and $a + 1$.

$V = (a - 1) \times a \times (a + 1) = a^3 - a$

We are seeking for a 4-digit number (a^3) that is just over 1000.

$10^3 = 1000$ and $11^3 = 1331$.

So $a = 11$. $V = a^3 - a = 11^3 - 11 = 1320$.

8. E.

Let C be the cost of a pencil in cents, N be the number of pencils each student bought, and S be the number of students who bought pencils. Then $C \cdot N \cdot S = 2015 = 5 \cdot 13 \cdot 31$, and $C > N > 1$. Because a majority of the students bought pencils, $40 \ge S > 40/2 = 20$. Therefore $S = 31$, $N = 5$, and $C = 13$.

9. A.

Method 1:

Let $\sqrt{6-\sqrt{11}} + \sqrt{6+\sqrt{11}} = x$ (1)

Squaring both sides of (1): $(\sqrt{6-\sqrt{11}} + \sqrt{6+\sqrt{11}})^2 = x^2 \quad \Rightarrow$

$(\sqrt{6-\sqrt{11}} + \sqrt{6+\sqrt{11}})^2 = 6-\sqrt{11}+2\sqrt{(6-\sqrt{11})(6+\sqrt{11})}+6+\sqrt{11}$

$= 12+2\sqrt{(6-\sqrt{11})(6+\sqrt{11})} = 12+2\sqrt{6^2-11} = 12+2\sqrt{25} = 12+10 = 22$.

$x^2 = 22 \qquad \Rightarrow \qquad x = \sqrt{22}$.

Method 2:

We see that $a = 6$, $b = 11$, and $a^2 - b = 6^2 - 11 = 25 = 5^2$. Therefore the given nested radical can be denested.

By the formula, we have

$$\sqrt{6+\sqrt{11}} = \sqrt{\frac{6+\sqrt{6^2-11}}{2}} + \sqrt{\frac{6-\sqrt{6^2-11}}{2}} = \sqrt{\frac{11}{2}} + \sqrt{\frac{1}{2}}.$$

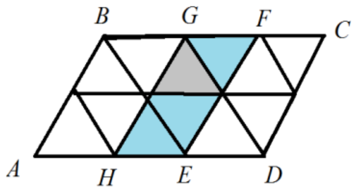
Similarly, $\sqrt{6-\sqrt{11}} = \sqrt{\frac{6+\sqrt{6^2-11}}{2}} - \sqrt{\frac{6-\sqrt{6^2-11}}{2}} = \sqrt{\frac{11}{2}} - \sqrt{\frac{1}{2}}$.

Thus $\sqrt{6-\sqrt{11}} + \sqrt{6+\sqrt{11}} = \sqrt{\frac{11}{2}} + \sqrt{\frac{1}{2}} + \sqrt{\frac{11}{2}} - \sqrt{\frac{1}{2}} = 2\sqrt{\frac{11}{2}} = 2\sqrt{\frac{2\times 11}{2\times 2}} = \sqrt{22}$.

10. C.
One complete rotation goes 3 + 4 + 6 + 2 + 7 = 22 unit length.
2015 = 22 × 91 + 13.
We only need to roll the pentagon 13 units. 3 + 4 + 6 = 13. Therefore *CD* will touch *x* = 2015 and be completely on the *x*-axis.

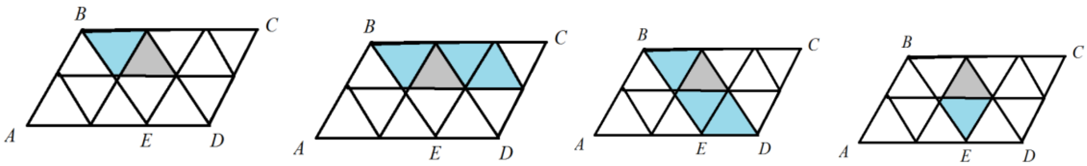
11. E.
We have $\binom{2}{1}\times\binom{2}{1}\times\binom{2}{1}\times\binom{1}{1}=8$

Parallelograms with sides parallel to *AB* and *BC*.



We have $\binom{1}{1}\times\binom{2}{1}\times\binom{2}{1}\times\binom{1}{1}=4$ more parallelograms with sides parallel to *BE* and *BC*.

Total number of parallelograms that contain the shaded triangle is 12.



12. A.
Method 1:
There are 5 · 25 = 125 painted faces all of which are equally likely. There are 125 · 6 = 750 faces altogether. Therefore the probability is 125/750 = 1/6.

Method 2:
Number of cubes painted 1 face: $12\times 4+9=57$.
Number of cubes painted 2 faces: $12\times 2+4=28$.
Number of cubes painted 3 faces: 4.
Number of cubes painted 0 side: 27 (We do not need this information).

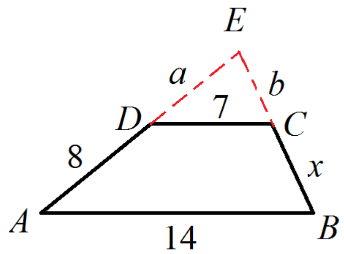
The probability is $P=\frac{57}{125}\times\frac{1}{6}+\frac{28}{125}\times\frac{2}{6}+\frac{4}{125}\times\frac{3}{6}=\frac{1}{6}$.

13. E.

$\frac{a}{7}=\frac{a+8}{14}=\frac{8}{7} \Rightarrow \quad a=8$.

$\frac{a}{b}=\frac{8}{x} \Rightarrow \quad b=x$.

By the triangle inequality theorem,



$AE-AB<EB<AE+AB \quad \Rightarrow$

$16-14<x+x<16+14 \Rightarrow \quad 2<2x<30$

$\Rightarrow 1<x<15 \quad \Rightarrow \quad 2\le x\le 14$.

The number of integer values for the length of the other leg is then 14 – 2 + 1 = 13.

14. A.

In the first 3 hours, each person works one hour and they finish $(\frac{1}{4}+\frac{1}{6}+\frac{1}{8})\times 1=\frac{13}{24}$ of the job.

The job left is $\frac{11}{24}$.

Let x be the number of hours Chandra works. In the next round, we have

$$\frac{1}{4}\times 1+\frac{1}{6}\times 1+\frac{1}{8}\times x=\frac{11}{24} \quad \Rightarrow \quad \frac{1}{8}\times x=\frac{11}{24}-\frac{1}{4}-\frac{1}{6}=\frac{11-6-4}{24}=\frac{1}{24}\Rightarrow \quad x=\frac{1}{3}$$

So Chandra paints the house $1+\frac{1}{3}=\frac{4}{3}$ hours.

15. D.

Method 1:

We find the time needed for each person to complete the path.

Alex needs $\frac{1000}{120}=\frac{25}{3}$ minutes.

Bob needs $\frac{1000}{80}=\frac{25}{2}$ minutes.

Charlie needs $\frac{1000}{70}=\frac{100}{7}$ minutes.

We now find the least common multiple of them.

$$LCM(\frac{a}{b},\frac{c}{d})=\frac{ac}{GCF(ad,bc)}=\frac{LCM(ad,bc)}{bd}$$

$$LCM(\frac{25}{3},\frac{25}{2})=\frac{25\times 25}{GCF(25\times 2,25\times 3)}=\frac{25\times 25}{25}=25$$

$$LCM(25,\frac{100}{7})=LCM(\frac{25}{1},\frac{100}{7})=\frac{25\times 100}{GCF(25\times 7,100\times 1)}=\frac{25\times 100}{25}=100$$

So at 11:40 a.m. they will first all be together again at point *A*.

Method 2:
Time needed for Alex to catch Bob is 1000 ÷ (120 – 80) = 25 minutes
Time needed for Alex to catch Charlie is 1000 ÷ (120 –70) = 20 minutes
Time needed for Bob to catch Charlie is 1000 ÷ (80 –70) = 100 minutes
LCM (25, 20, 10) = 100.
So at 11:40 a.m. they will first all be together again at point *A*.

16. E.
A compatible group will either exclude all these three animals or include exactly one of them. This can be done in $\binom{9}{6}+\binom{3}{1}\binom{9}{5}=84+378=462$ committees.

17. A.
The remainder is 1 when 3^n is divided by 8, where n is even. So the sum of every 8 terms will have a remainder of 0 when divided by 8.
2014/2 = 1007 = 121 × 8 + 7. So the remainder is 7 when $3^2 + 3^4 + \cdot\cdot\cdot + 3^{2014}$ is divided by 8.

The remainder is 3 when 3^m is divided by 8, where m is odd. So the sum of every 8 terms will have a remainder of 0 when divided by 8.
(2015 – 1)/2 + 1 = 1008 = 126 × 8 + 0. So the remainder is 0 when $3^1 + 3^3 + \cdot\cdot\cdot + 3^{2015}$ is divided by 8.
So the required remainder is 7 + 0 = 7.

18. B.
Let x be the amount of antifreeze to be drained off.

Name	C	V	S
A_1	0.4	9	3.6
	0.4	x	$0.4x$
A_2	0.4	$9-x$	$0.4(9-x)$
B	1.0	x	$1.0x$
Mixture	0.7	9	0.7(9)

$0.4(9-x) + 1.0x = 0.7(9) \quad \Rightarrow \quad x = 4.5$ liters.

19. C.

The four-digit positive integer can be written as $1000a + 100b + 10c + d$, where $a, b, c,$ and d are digits.

We have $1000a + 100b + 10c + d + a + b + c + d = 2015 \quad \Rightarrow$

$1001a + 101b + 11c + 2d = 2015$.

Case 1: $a = 2$.

$1001a + 101b + 11c + 2d = 2015 \quad \Rightarrow \quad 101b + 11c + 2d = 2015 - 2002 = 13$.

So $b = 0$, $c = 1$, and $d = 1$.

The four-digit positive integer is 2011.

Case 2: $a = 1$.

$1001a + 101b + 11c + 2d = 2015 \quad \Rightarrow \quad 101b + 11c + 2d = 2015 - 1001 = 1014$.

So $b = 9$, $c = 9$, and $2d = 1014 - 909 - 99 = (105 - 99) = 6$. So $d = 3$.

The four-digit positive integer is 1993.

The answer is 2011 + 1993 = 4004.

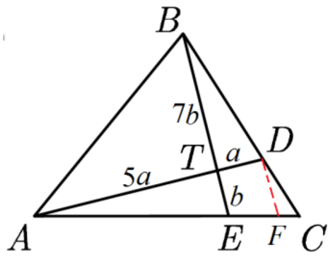
20. C.

Pipe A works at a rate of $\frac{1}{6}$ (tank/hour) and pipe B works at a rate of $\frac{1}{4}$. Working together, they fill the tank at a rate of $\frac{1}{6}+\frac{1}{4}$. Let x be the time needed to fill the tank if pipes A and B worked together.

$x(\frac{1}{6}+\frac{1}{4}) = 1 \Rightarrow x = 2$ hours.

Since pipe C will fill the tank in the same number of hours that it will take pipes A and B working together to fill the tank, Ppipe C works at a rate of $\frac{1}{2}$. When all three pipes work together, they fill the tank at a rate of $\frac{1}{6}+\frac{1}{4}+\frac{1}{2}$. Working together for one hour, they fill $1\times(\frac{1}{\frac{1}{6}+\frac{1}{4}+\frac{1}{2}})=\frac{10}{12}=\frac{5}{6}$ of the tank.

21. A.
Method 1:
Draw $DF \parallel BE$.



$\Delta ADF \sim \Delta ATE$. $\frac{AD}{AT}=\frac{DF}{TE} \Rightarrow \frac{6a}{5a}=\frac{DF}{b} \Rightarrow DF=\frac{6b}{5}$

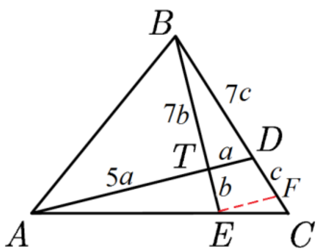
$\Delta CBE \sim \Delta CDF$. $\frac{BE}{DF}=\frac{BC}{CD} \Rightarrow \frac{8b}{DF}=\frac{BC}{CD}$

$\Rightarrow \frac{8b}{\frac{6b}{5}}=\frac{BC}{CD} \Rightarrow \frac{20}{3}=\frac{BC}{CD}$

$\Rightarrow \frac{20}{3}=\frac{BD+CD}{CD} \Rightarrow \frac{20}{3}=\frac{BD}{CD}+1$

$\Rightarrow \frac{20}{3}-1=\frac{BD}{CD}=\frac{17}{3} \Rightarrow \frac{CD}{BD}=\frac{3}{17}$.

Method 2:
Draw $EF \parallel AD$.



$\Delta BEF \sim \Delta BTD$. $\frac{BE}{BT}=\frac{EF}{TD} \Rightarrow \frac{8b}{7b}=\frac{EF}{a} \Rightarrow EF=\frac{8a}{7}$

$\Delta ACD \sim \Delta EFC$. $\frac{AD}{EF}=\frac{CD}{CF} \Rightarrow \frac{6a}{\frac{8a}{7}}=\frac{CD}{CF}$

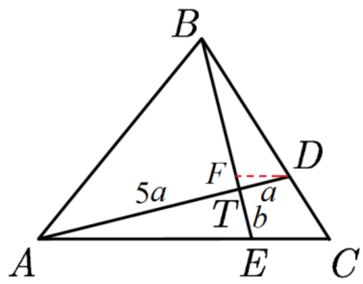
$\Rightarrow \frac{CF}{CD}=\frac{4}{21} \Rightarrow \frac{4}{21}=\frac{CD-DF}{CD}=1-\frac{DF}{CD}$

$\Rightarrow 1-\frac{4}{21}=\frac{DF}{CD}=\frac{17}{21} \Rightarrow CD=\frac{21}{17}DF$

$$\frac{CD}{BD}=\frac{\frac{21}{17}DF}{7DF}=\frac{3}{17}.$$

Method 3:

Draw $DF \,//\, AC$.



$\Delta AET \sim \Delta DFT.$ $\frac{AT}{DT}=\frac{TE}{TF}\Rightarrow\frac{5a}{a}=\frac{b}{TF}\Rightarrow \quad TF=\frac{b}{5}$

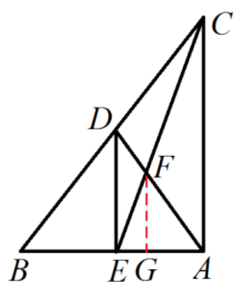
$\Delta BEC \sim \Delta BFD.$ $\frac{BE}{BF}=\frac{BC}{BD}\Rightarrow \quad \frac{8b}{BT-TF}=\frac{BC}{BD}$

$\Rightarrow \quad \frac{8b}{7b-\frac{b}{5}}=\frac{BC}{BD}\Rightarrow \quad \frac{8b}{\frac{34}{5}b}=\frac{BD-CD}{BD}$

$\Rightarrow \quad \frac{20}{17}=1-\frac{CD}{BD}\Rightarrow \quad \frac{CD}{BD}=\frac{20}{17}-1=\frac{3}{17}$

22. A.

Method 1:



Draw $FG\perp AB$.

$FG \,//\, DE \,//\, AC$.

$\frac{1}{DE}+\frac{1}{AC}=\frac{1}{FG} \quad \Rightarrow \quad \frac{1}{\frac{1}{2}AC}+\frac{1}{AC}=\frac{1}{FG}$

$\Rightarrow \quad \frac{3}{AC}=\frac{1}{FG}\Rightarrow \quad FG=\frac{AC}{3}.$

The area of triangle AEF is $\frac{1}{2}AE\times FG=\frac{1}{2}\times\frac{1}{2}AB\times\frac{AC}{3}=\frac{1}{6}\times(\frac{1}{2}AB\times AC)$

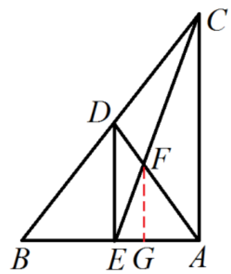
$=\frac{1}{6}S_{\Delta ABC}=\frac{1}{6}\times 360=60$

Method 2:

Draw $FG\perp AB$.

$FG \,//\, DE \,//\, AC$.

$$\frac{FG}{\frac{1}{2}AC}=\frac{FG}{DE}=\frac{AG}{AE} \qquad (1)$$



$$\frac{FG}{AC} = \frac{EG}{AE} \qquad (2)$$

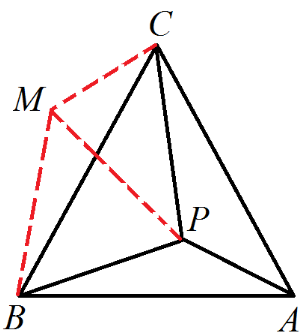
(1) + (2): $\frac{3FG}{AC} = \frac{AE}{AE} = 1 \quad \Rightarrow \quad FG = \frac{AC}{3}$.

The area of triangle AEF is

$$\frac{1}{2}AE \times FG = \frac{1}{2} \times \frac{1}{2}AB \times \frac{AC}{3} = \frac{1}{6} \times (\frac{1}{2}AB \times AC) = \frac{1}{6}S_{\Delta ABC} = \frac{1}{6} \times 360 = 60.$$

23. A.

We rotate ΔBAP anti clock wise 60° such that BA and BC overlap. PB will be in the position of BM, and PA will be in the position of MC. So $BM = BP$, $MC = PA$, $\angle PBM = 60°$.



Thus ΔBPM is an equilateral triangle.

Therefore $PM = PB = 2\sqrt{3}$.

In ΔMCP, $PC = 4$. $MC = PA = 2$, $PM = 2\sqrt{3}$. We see that $PC^2 = PM^2 + MC^2$, and $PC = 2MC$.

So ΔMCP is a right triangle with $\angle CMP = 90°$ and $\angle CPM = 30°$.

We also know that ΔPBM is an equilateral triangle with $\angle BPM = 60°$. Then we know that ΔBPC is a right triangle with $\angle BPC = 90°$.

Thus $BC^2 = BP^2 + PC^2 = (2\sqrt{3})^2 + 4^2 = 28 \Rightarrow BC = 2\sqrt{7}$.

The area of triangle ABC is $\frac{BC^2}{4}\sqrt{3} = \frac{(2\sqrt{7})^2}{4}\sqrt{3} = 7\sqrt{3}$.

24. B.

Case 1: The middle card, showing the spade, does not have a 3 printed on the other side.

There are four possible ways for the back of the middle card: 1, 2, 4, or 5.
There are 4 possible ways for the back of the left side card: 3/club, 3/diamond, 3/heart or 3/spade. There are 3 possible ways for the back of the right side card since one is already taken by the left side card.

There are 4 × 4 × 3 = 48 different ways for this scenario to occur.

Among them, there are 1 × 4 × 3 = 12 ways such that the left side card has the spade on the back, and 3 × 4 × 1 = 12 ways such that the right side card has the spade on the back. That is, we have 24 ways that one of the cards showing the number 3 has a spade printed on the other side.

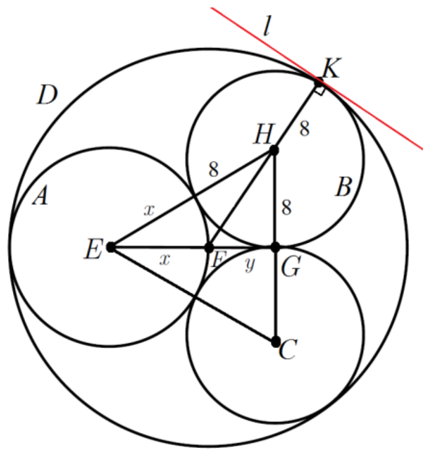
Case 2: The middle card, showing the spade, has a 3 printed on the other side.

There is only one way for the back of the middle card: 3/spade. There are 3 possible ways for the back of the left side card: 3/club, 3/diamond, or 3/heart. There are 2 possible ways for the back of the right side card since two are taken by the left side card and the middle card.

There are 3 × 1 × 2 = 6 different ways for this scenario to occur. But none of these scenarios has a spade printed on the other side of one of the cards showing the number 3. So of the 48 + 6 = 54 possible scenarios, the probability that one of the cards showing the number 3 has a spade printed on the other side is 24/54 = 4/9.

25. D.
Let *E, H*, and *F* be the centers of circles *A*, *B*, and *D*, respectively, and let *G* be the point of tangency of circles *B* and *C*.
Connect *EH, EC, CH,* and *EG*.
Since circles *B* and *C* are congruent, we know that *EG* is the perpendicular bisector of *HC*.

Let $x = EF$ and $y = FG$.



Since the center of circle *D* lies on circle *A* and the circles have a common point of tangency, the radius of circle *D* is $2x$, which is the diameter of circle *A*.

Draw the line *l* tangent to circles *D* and *B* at *K*.
Connect *KF* and we know that *KF* goes through *H*.

Applying the Pythagorean Theorem to right triangles *EGH* and *FGH* gives:

$$(x+8)^2-(x+y)^2=8^2 \qquad (1)$$

$(2x-8)^2 - y^2 = 8^2$ (2)

(1) can be simplified to $y^2 + 2xy - 16x = 0$ (3)

(2) can be simplified to $4x^2 - 32x - y^2 = 0$ (4)

(3) + (4): $4x^2 - 48x + 2xy = 0$ $\Rightarrow$ $2x^2 - 24x + xy = 0$ $\Rightarrow$ $x(2x - 24 + y) = 0$.

We know tha x ≠ 0. So we have $2x - 24 + y = 0$ $\Rightarrow$ $y = 24 - 2x$ (5)

Subsituting (5) into (4): $4x^2 - 32x - (24 - 2x)^2 = 0$

$\Rightarrow$ $4x^2 - 32x - 24^2 - 4x^2 + 2 \times 2 \times 24x = 0$ $\Rightarrow$ $64x - 24^2 = 0$ $\Rightarrow$ $x = 9$.