Problem Solving Uisng Vieta's Theorem Sample Problems

Example 1. If 2 is a root of $x^2 - x - 2 = 0$, find the second root.

Example 2. The sum of two real numbers is 10 and the product of them is 22. Find the larger of the two numbers.

Example 3. What is the value of m such that one root is the reciprocal of the second root in the equation $8x^2 - (2m + 1)x + m - 7 = 0$.

A. 1   B. 15   C. −1   D. 7   E. 2

Example 4. (2002 AMC 10 B) Suppose that a and b are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions a and b. What is the pair (a, b)?

(A) (−2, 1)   (B) (−1, 2)   (C) (1, −2)   (D) (2, −1)   (E) (4, 4)

Example 5. The nonzero roots of the equation $x^2 + 6x + k = 0$ are in the ratio 2:1. What is the value of k?

Example 6. What is the sum of the reciprocals of the solutions of $4x^2 - 13x + 3 = 0$? Express your answer as a common fraction.

Example 7. (2012 Mathcounts Handbook Problem 256) The solutions $x = u$ and $x = v$ of the quadratic equation $rx^2 + sx + t = 0$ are reciprocals of the solutions of the quadratic equation $(2 + a)x^2 + 5x + (2 - a) = 0$ for some integer a. If the *GCF* of r, s and t is 1, what is the value of $r + s + t$?

Example 8. (Mathcounts) What is the average value of the three roots of the equation $x^3 - 12x^2 - 4x + 48 = 0$?

Example 9. (2015 AMC 12A Problem 18, AMC 10A Problem 23) The zeros of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of a?

(A) 7   (B) 8   (C) 16   (D) 17   (E) 18

Example 10. (AMC) If r and s are the roots of $x^2 - px + q = 0$, then $r^2 + s^2$ equals:
(A) $p^2 + 2q$ (B) $p^2 - 2q$ (C) $p^2 + q^2$ (D) $p^2 - q^2$ (E) p^2

Example 11. Find $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ if α and β are two roots of the equation $x^2 - x - 3 = 0$.

Example 12. Let α and β be the real roots of $x^2 + 2x - 5 = 0$. Compute $\alpha^2 + \alpha\beta + 2\alpha$.

Example 13. (1996 China Middle School Math Competition) Let x_1 and x_2 be the roots of the equation $x^2 + x - 3 = 0$. What is value of $x_1^3 - 4x_2^2 + 19$?
(A) -4 (B) 8 (C) 6 (D) 0 (E) 8

Example 14. Solve: $\begin{cases} 2x + 3y = 8 \\ xy = 2 \end{cases}$

Example 15. (AMC) How many distinct ordered triples (x, y, z) satisfy the equations

$$x + 2y + 4z = 12$$
$$xy + 4yz + 2xz = 22$$
$$xyz = 6?$$

(A) none (B) 1 (C) 2 (D) 4 (E) 6

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Keys:

1. **Solution:** -1
2. **Solution:** $5+\sqrt{3}$.
3. **Solution:** B.
4. **Solution:** (C).
5. **Solution:** 8.
6. **Solution:** $\frac{13}{3}$.
7. **Solution:** 9.
8. **Solution:** 4.
9. **Solution:** (C)
10. **Solution:** (B).
11. **Solution:** $\frac{7}{9}$.
12. **Solution:** 0.
13. **Solution:** (D).
14. **Solution:** (1, 2), and (3, $\frac{2}{3}$).
15. **Solution:** (E).