BASIC KNOWLEDGE

1. Terms

A permutation is an arrangement or a listing of things in which order is important.

A combination is an arrangement or a listing of things in which order is not important

2. Definition

The symbol ! (factorial) is defined as follows:

$$0! = 1,$$

and for integers $n \geq 1$,

$$n! = n \cdot (n-1) \cdots 1.$$

$1! = 1$,
$2! = 2 \cdot 1=2$,
$3! = 3 \cdot 2 \cdot 1=6$,
$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$,
$5! = 5 \cdot 4 \cdot 3 \cdot 1 \cdot 1 = 120$,
$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$.

3. Permutations

(1). Different elements, with no repetition. Take r elements each time from n distinct elements ($1 \leq r \leq n$).

Number of permutations $P(n,r) = \dfrac{n!}{(n-r)!}$ (1)

(2). n distinct objects can be permutated in n! permutations.

We let $n = r$ in (1) to get $P(n, n) = n!$ (2)

Proof of (2):

The first object can be chosen in n ways, the second object in $n - 1$ ways, the third in $n - 2$, etc. By the Fundamental Counting Principle, we have
$n(n - 1)(n - 2) \cdots 2 \cdot 1 = n!$ ways.

Example 1. In how many ways can the letters of the word MATH be arranged? (Mathcounts Handbooks)

Solution: 24.
$4! = 4 \times 3 \times 2 \times 1 = 24$.

(MATH MAHT MTAH MTHA MHTA MHAT AMTH AMHT ATMH ATHM AHTM AHMT TAMH TAHM TMAH TMHA THMA THAM HATM HAMT HTAM HTMA HMTA HMAT)

Example 2: In how many different orders can a group of six people be seated in a row of 6 seats?

Solution: 24.
$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

Try it yourself:

(1). In how many ways can the letters of the word COUNTS be arranged?

Answer: 720 (ways)

(2). How many different four-digit, positive integers are there where each digit is a prime number? No digit is allowed to be used twice in any integer.

Answer: 24(integers)

4. Grouping

THEOREM: Let the number of different objects be n. Divide n into r groups $A_1, A_2, \ldots, A_r$ such that there are n_1 objects in group A_1, n_2 objects in group A_2, ..., n_r objects in the group A_r, where $n_1 + n_2 + \cdots + n_r = n$. The number of ways to do so is

$$\frac{n!}{n_1!n_2!\cdots n_r!} \qquad (3)$$

Proof:

(1). There are $\binom{n}{n_1}$ ways to take out n_1 elements from n elements to form group A_1.

(2). There are $\binom{n-n_1}{n_2}$ ways to take out n_2 elements from $n - n_1$ elements to form group A_2.

(3). Continue the process until there are n_r elements left to form group A_r.

(4). The total number of ways, based on the Fundamental Counting Principle, is

$$\binom{n}{n_1}\binom{n-n_1}{n_2}\cdots\binom{n_r}{n_r} = \frac{n!}{n_1!n_2!\cdots n_r!}$$

Example 1: In how many ways may we distribute 12 books to Alex, Bob, Catherine, and Denise such that each person gets 3 books?

Solution:

$$\frac{n!}{n_1!n_2!\cdots n_r!} = \frac{12!}{3!3!3!3!} = \frac{12!}{(3!)^4} = 369600$$

Example 2: In how many ways may we distribute 12 books to Alex, Bob, Catherine, and Denise such that Alex and Bob each get 4 books and Catherine and Denise each get 2 books?

Solution: 3207900.

$$\frac{n!}{n_1!n_2!\cdots n_r!} = \frac{12!}{3!3!2!2!} = 3207900$$

Example 3: In how many ways may we distribute 12 books to Alex, Bob, Catherine, and Denise such that Alex gets 5 books, Bob each get 4 books, Catherine gets 3 books, and Denise gets 2 books?

Solution: 83160

$$\frac{n!}{n_1!n_2!\cdots n_r!} = \frac{12!}{5!4!2!1!} = 83160$$

THEOREM: Let there be r types of objects: n_1 of type 1, n_2 of type 2; etc. The number of ways in which these $n_1 + n_2 + \cdots + n_r = n$ objects can be rearranged is

$$\frac{n!}{n_1!n_2!\cdots n_r!} \qquad (4)$$

The proof is the same as the proof for (3)

Example 1. In how many ways may the letters of the word *Mississippi* be permutated?
Solution:
The word Mississippi has 4 i's, 4 s's, 2 p's, and 1 m.

$$\frac{n!}{n_1!n_2!\cdots n_r!} = \frac{11!}{4!4!2!1!} = 34650$$

Example 2. In how many ways may the letters of the word *Mississippi* be permutated in such a way that two p's are not next to each other?

Solution: 28350.
We tie two p's together and think of it as one letter and subtract the number of ways the resulting word can be permutated from 34650, the total number of ways the original word can be permutated.

The number of permutations of the 10 letters is

$$\frac{n!}{n_1!n_2!\cdots n_r!} = \frac{10!}{4!4!1!1!} = 6300$$

The number of permutations sought is
34650 – 6300 =28350.

Example 3. In how many ways may the letters of the word *MASSACHUSETTS* be permutated?

Solution: 64864800.

There are now 13 distinguishable objects, which can be permuted in 13! different ways. However, we must divide this accordingly.
AA, *SSSS*, *TT*, *M*, *C*, *H*, *E*, *U*.

$$N = \frac{13!}{2!\ 4!\ 2!\ 1!\ 1!\ 1!\ 1!\ \ 1!} = \frac{13!}{2!\ 4!\ 2!} = 64864800.$$

Example 4. In how many ways may we permute the letters of the word MASSACHUSETTS in such a way that *MATH* is always together, in this order?

Solution: 151200
We tie the four letters *MATH* together and consider them as one letter with the remaining 9 letters: *SSSS*, *A*, *C*, *U*, *E*, and *T*.

The total number of permutations is

$$N = \frac{10!}{4!\ 1!\ 1!\ 1!\ 1!\ 1!\ \ 1!} = \frac{10!}{4!} = 151200$$

Try it yourself:

(1). How many distinguishable arrangements are possible using all of the letters of BREEZE? (Mathcounts Competitions)

Answer: 120(arrangements)

(2). How many different arrangements are there using all of the letters in the word PARALLEL? (Mathcounts Competitions)

Answer: 3,360(arrangements)

(3). How many distinct ways can the letters of the word PEOPLE be arranged so that the two P’s are together and the two E’s are together? (Mathcounts Competitions)

Answer: 24

(4). How many different arrangements of the six letters of the word "YELLOW" can be made if the first letter must be "W" and the last letter must be "L"? (Mathcounts Competitions)

Answer: 24(arrangements)

THEOREM: For any positive integer n, the multinomial expansion of

$(x+y+z+....+w)^n$

is the sum of all terms of the form

$$\frac{n!}{n_1!n_2!\cdots n_r!}x^{n_1}y^{n_2}z^{n_3}....w^{n_r} \qquad (5)$$

where $n_1 + n_2 + \cdots + n_r = n$.

Example 1. What is the coefficient of x^2y^6 in the expansion of $(x + y)^8$?

Solution: 28.

$$\frac{n!}{n_1!n_2!\cdots n_r!} = \frac{8!}{2!\,6!} = 28.$$

Example 2. What is the coefficient of x^7y^3 in the expansion of $(x + y)^{10}$?

Solution: 120.

$$\frac{n!}{n_1!n_2!\cdots n_r!} = \frac{10!}{3!\,7!} = \binom{10}{3} = 120.$$

Example 3. What is the coefficient of x^7y^3 in the expansion of $(x + 2y)^{10}$?

Solution: 120.

$$\frac{n!}{n_1!n_2!\cdots n_r!}x^{n_1}y^{n_2}z^{n_3}....w^{n_r} = \frac{10!}{3!7!}x^7(2y)^3$$

The coefficient is $\frac{10!}{3!7!}2^3 = 120 \times 8 = 960$.

Example 4. When $(x + 2y - z)^8$ is expanded, and the like terms are combined, what is the coefficient of the term $x^3y^2z^3$? (Mathcounts Competition 2009 National Sprint Round 29)

Solution: –2240

We know that $\frac{n!}{n_1!n_2!\cdots n_r!} x^{n_1} y^{n_2} z^{n_3} \ldots w^{nr}$

$$= \frac{8!}{3!2!3!} x^3 (2y)^2 (-z)^3 = -2240 x^3 y^2 z^3$$

The coefficient of the term $x^3y^2z^3$ is –2240.

Try it yourself

(1). What is the coefficient of x^3y^7 in the expansion of $(x + y)^{10}$?
Answer: 120.

(2). What is the coefficient of $x^2y^3z^2w^3$ in the expansion of $(x + y + z + w)^{10}$?
Answer: 25200.

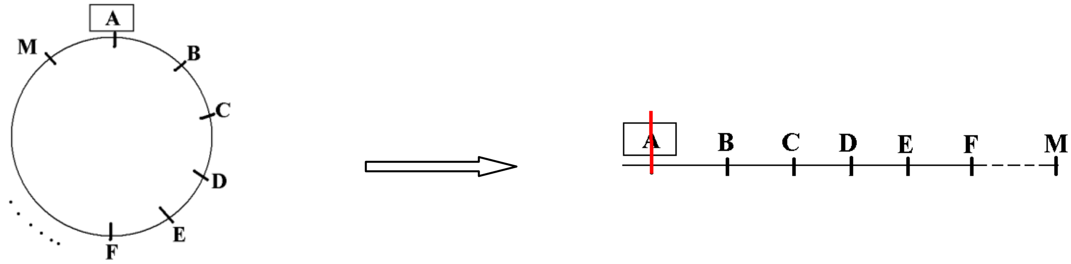
(3). What is the coefficient of $x^2y^3z^7$ in the expansion of $(x + y + z)^{12}$?
Answer: 7920.

5. Circular Permutations

The number of circular permutations (arrangements in a circle) of n distinct objects is

$$(n - 1)! \qquad (6)$$

We can think of this as n people being seated at a round table. Since a rotation of the table does not change an arrangement, we can put person A in one fixed place and then consider the number of ways to seat all the others. Person B can be treated as the first person to seat and M the last person to seat. The number of ways to arrange persons A to M is the same as the number of ways to arrange persons B to M in a row. So the number of ways is $(n - 1)!$.



Example 1. In how many ways is it possible to seat seven people at a round table?

Solution:
$(n - 1)! = (7 - 1)! = 6! = 720$.

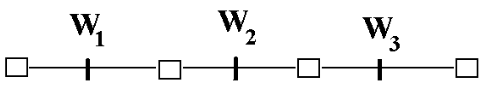
Example 2. In how many ways is it possible to seat seven people at a round table if Alex and Bob must not sit in adjacent seats?

Solution: 600.
The number of ways to sit 7 people at a round table is 720.

We find the number of ways Alex and Bob sit together by seeing them as a unit. There are $(6 - 1)! = 5!$ ways. The result is multiplied by 2 if we alter Alex's and Bob's positions. The solution is then $720 - 2 \times 5! = 720 - 120 = 600$.

Example 3. In how many ways can four men and four women be seated at a round table if no two men are to be in adjacent seats?

Solution: 144.
We seat four women first. There are $(4 - 1)! = 3!$ ways to sit them. After the ladies are seated, we have 4! ways to seat four men in the small rectangles as shown in the figure below. $4! \times 3! = 144$.

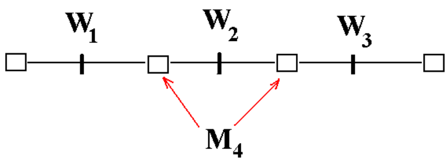


Example 4. In how many ways can four married couples be seated at a round table if no two men, as well as no husband and wife are to be in adjacent seats?

Solution: 12.

We already know from the last problem that there are (4 – 1)! = 3! to seat four women. After the ladies are seated, person M_4 (whose wife is not shown in the figure below) has two ways to sit. After he is seated in any one of the two possible seats, the other men have only one way to sit in the remaining steats.

The solution is 3! × 2 = 12.



Example 5. In how many ways can a family of six people be seated at a round table if the youngest kid must sit between the parents?

Solution: 12.

We link two parents and the youngest kid together to form a unit. There are (4 – 1)! ways to seat them at the table. The result must be multiplied by 2 since we can switch the positions of the two parents. The solution is (4 – 1)! × 2 = 12.

Try it yourself:

(1). In how many different orders can a group of six people be seated around a round table? (Mathcounts Competitions)

Answer: 120(orders)

(2). In how many distinguishable ways can four identical red chips and two identical white chips be arranged in a circle? (Mathcounts Competitions)

Answer: 3(ways)

(3). In how many ways can five men and five women be seated at a round table if no two men are to be in adjacent seats?

Answer: 2880.

(4). In how many ways can a family of seven people be seated at a round table if the youngster kid must sit between the parents?

Solution: 48.

6. Combinations

Definition Let *n, r* be non-negative integers such that $0 \le r \le n$. The symbol $\binom{n}{r}$ (read "*n* choose *m*") is defined and denoted by

$$\binom{n}{r} = \frac{P(n,r)}{P(r,r)} = \frac{n!}{r!(n-r)!} \qquad (7)$$

Remember: $\binom{n}{0} = 1$, $\binom{n}{1} = n$, and $\binom{n}{n} = 1$

Since $n - (n - r) = r$, we have $\binom{n}{r} = \binom{n}{n-r}$ (8)

Examples.

$\binom{2}{2} = \frac{2\times 1}{2\times 1} = 1\times 1 = 1$, $\binom{3}{2} = \frac{3\times 2}{2\times 1} = 3\times 1 = 3$,

$\binom{4}{2} = \frac{4\times 3}{2\times 1} = 2\times 3 = 6$

$\binom{5}{2} = \frac{5\times 4}{2\times 1} = 5\times 2 = 10$, $\binom{6}{2} = \frac{6\times 5}{2\times 1} = 3\times 5 = 15$, $\binom{7}{2} = \frac{7\times 6}{2\times 1} = 7\times 3 = 21$

$\binom{8}{2} = \frac{8\times 7}{2\times 1} = 4\times 7 = 28$, $\binom{9}{2} = \frac{9\times 8}{2\times 1} = 9\times 4 = 36$, $\binom{10}{2} = \frac{10\times 9}{2\times 1} = 5\times 9 = 45$

$\binom{8}{6} = \binom{8}{2} = 28$, $\binom{9}{7} = \binom{9}{2} = 36$, $\binom{10}{8} = \binom{10}{2} = 45$

Unlike Permutations, Combinations are used when order does not matter. If we have *n* different elements, and it doesn't matter which order we take the elements, the number of ways to take *m* elements where $1 \le m \le n$, is $\binom{n}{m}$

Example 1. Four people are selected from a group of 10 people to form a committee. How many ways are there?

Solution: 210 ways.
From a group of 10 people, we can choose a committee of 4 in
$\binom{10}{4}$ = 210 ways.

Example 2. In a group of 2 cats, 3 dogs, and 10 pigs in how many ways can we choose a committee of 6 animals if
(a) there are no constraints in species?
(b) the two cats must be included?
(c) the two cats must be excluded?
(d) there must be at least 3 pigs?
(e) there must be at most 2 pigs?
(f) Joe Cat, Billy Dog and Samuel Pig hate each other and they will not work in the same group. How many compatible committees are there?

Solution:
(a) There are 2 + 3 + 10 = 15 animals and we must choose 6. The order does not matter. Thus, there are
$\binom{15}{6}$ = 5005 6 possible committees.

(b) Since the 2 cats must be included, we must choose 6 − 2 = 4 more animals from a list of 15 − 2 = 13 animals, so there are $\binom{13}{4}$ = 715 4 possible committees.

(c) Since the 2 cats must be excluded, we must choose 6 animals from a list of 15 − 2 = 13, so there are $\binom{13}{6}$ = 1716 6 possible committees.
(d) If k pigs are chosen from the 10 pigs, $6-k$ animals must be chosen from the remaining 5 animals, so there are $\binom{10}{3}\binom{5}{3}+\binom{10}{4}\binom{5}{2}+\binom{10}{5}\binom{5}{1}+\binom{10}{6}\binom{5}{0}$ = 4770 committees.

(e) Observe that there cannot be 0 pigs, since that would mean choosing 6 other animals from the remaining animals which produces no ways. Hence, there must be either 1 or 2 pigs, and so 3 or 4 of the other animals. The total number is thus

$$\binom{10}{2}\binom{5}{4}+\binom{10}{1}\binom{5}{5}=235$$

(f) A compatible group will either exclude all these three animals or include exactly one of them. This can be done in $\binom{12}{6}+\binom{3}{1}\binom{12}{5}=3300$ committees.

Example 3. Consider the set of 5-digit positive integers.

(a) How many are there?
(b) How many do not have a 7 in their decimal representation?
(c) How many have at least one 7 in their decimal representation?

Solution:
(a) There are 9 possible choices (1, 2, 3, 4, 5, 6, 7, 8, 9) for the first digit and 10 possible choices (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) for the remaining digits. The number of choices is thus $9 \bullet 10^4 = 90000$.

(b) There are 8 possible choices (1, 2, 3, 4, 5, 6, ~~7~~, 8, 9) for the first digit and 9 possible choices (0, 1, 2, 3, 4, 5, 6, ~~7~~, 8, 9) for the remaining digits. The number of choices is thus $8 \bullet 9^4 = 52488$.

(c) The difference of the total number of 15-digit positive integers and the number of 5-digit integers that do not have a 7 in their decimal representations: 90000 − 52488 = 37512.

7. Combinations with Repetitions

THEOREM: *n* identical balls are put into *r* labeled boxes and the number of balls in each box is not limited. The number of ways is

$$\binom{n+r-1}{n} \text{ or } \binom{n+r-1}{r-1} \qquad (9)$$

Proof: Put r labeled boxes next to each other as shown in the figure below. Put n balls into these boxes. Now we take apart the top and bottom sides of the each box and the two sides of the two boxes at the end. The problem becomes finding the number of ways to permutate n identical balls with $r – 1$ identical partitions: $\frac{(n+r-1)!}{n!(r-1)!}$ or $\binom{n+r-1}{n}$ or $\binom{n+r-1}{r-1}$.

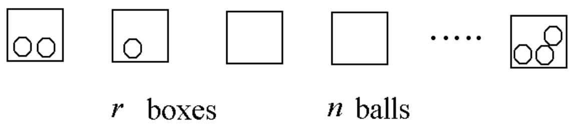


Figure 1

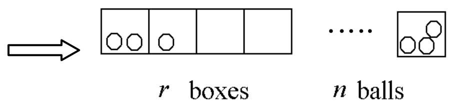


Figure 2

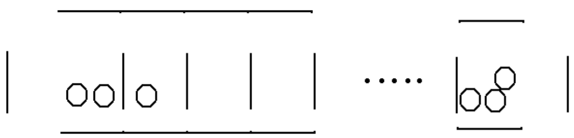


Figure 3

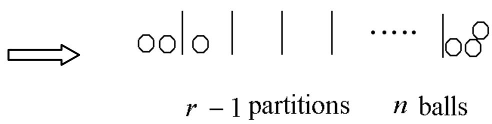


Figure 4

THEOREM: The number of terms in the expansion of $(x_1 + x_2 + x_3 + ... + x_r)^n$, after the like terms combined, is

$$\binom{n+r-1}{n} \text{ or } \binom{n+r-1}{r-1} \qquad (9)$$

Example 1. How many terms are there in the expansion of $(x+y+z)^4$ after all the like terms are combined?

Solution: 15.
The number of terms in the expansion of $(x+y+z)^4$ is the number of nonnegative integer solutions of $x+y+z=4$.

$\binom{n+r-1}{r-1} = \binom{4+3-1}{3-1} = \binom{6}{2} = 15$.

Example 2. How many terms are there in the expansion of $(a+b+c+d+ed+f)^4$ after all the like terms are combined?

Solution: 126.
This problem is the same as selecting 4 letters from six letter (*a, b, c, d, e,* and *f*) with repetition allowed.

$$\binom{n+r-1}{r-1} = \binom{4+6-1}{6-1} = \binom{9}{5} = 126$$

Try it yourself:

(1). How many terms are there in the expansion of $(x+y+z)^5$ after all the like terms are combined?

Answer: 21.

(2). How many terms are there in the expansion of $(x+y+z)^n$ after all the like terms are combined?
Answer: $\frac{1}{2}(n+1)(n+2)$

THEOREM: Let *n* be a positive integer. The number of positive integer solutions to
$x_1 + x_2 + \cdots + x_r = n$
is $\binom{n-1}{r-1}$. (10)

Proof. Write n as $n = 1+1+ \cdots +1+1$, where there are n 1's and $n - 1$ plus signs. In order to decompose n in r summands, we choose $r - 1$ plus signs from the $n - 1$, that is $\binom{n-1}{r-1}$.

Example 1. How many positive integral solutions are there to the equation $x_1 + x_2 + \cdots + x_6 = 12$?

Solution: 462.

$$\binom{n-1}{r-1} = \binom{12-1}{6-1} = \binom{11}{5} = 462$$

Example 2. In how many ways may we write the number 9 as the sum of three positive integer summands? Here order counts, so, for example, 1 + 7 + 1 is to be regarded different from 7 + 1 + 1.

Solution: 28.
We are seeking integral solutions to a + b + c =9, a > 0, b > 0, c > 0. The number of solutions is thus

$$\binom{n-1}{r-1} = \binom{9-1}{2-1} = \binom{8}{2} = 28$$

Example 3. In how many ways can 10 be written as the sum of four positive integer summands?
Solution:

$$\binom{n-1}{r-1} = \binom{10-1}{4-1} = \binom{9}{3} = 84$$

Try it yourself:

(1). How many positive integral solutions are there to the equation $x + y + z = 11$?

Answer: 45.

(2). How many positive integral solutions are there to the equation $x + y + z = n$?

Answer: $\frac{1}{2}(n-1)(n-2)$

THEOREM: Let n be a positive integer. The number of non-negative integer solutions to $y_1 + y_2 + \cdots + y_r = n$ is

$$\binom{n+r-1}{n} \text{ or } \binom{n+r-1}{r-1} \qquad (9)$$

Proof. Set $x_r - 1 = y_r$. Then $x_r \geq 1$. The equation $x_1 + x_2 + \cdots + x_r = n$ is equivalent to $x_1 + x_2 + \cdots + x_r = n + r$, which has $\binom{n+r-1}{r-1}$ solutions.

Example 1: How many nonnegative integral solutions are there to the equation $x_1 + x_2 + \cdots + x_6 = 12$?

Solution: 6188.

$$\binom{n+r-1}{r-1} = \binom{12+6-1}{6-1} = \binom{17}{5} = 6188$$

Try it yourself:

(1). How many nonnegative integral solutions are there to the equation $x + y + z = 11$?

Answer: 78

(2). How many nonnegative integral solutions are there to the equation $x + y + z = n$?

Answer: $\frac{1}{2}(n+1)(n+2)$

EXERCISES

Problem 1. A committee of 3 teachers is to be selected from a group of 10 teachers to write next year's MATHCOUNTS competition. How many different committees can be selected? (Mathcounts Competitions)

Problem 2. If no one shares an office, in how many ways can 3 people be assigned to 5 different offices? (Mathcounts Competitions)

Problem 3. A four-letter sequence is formed by rearranging the letters in the word "snow". How many different four-letter sequences are possible? (Mathcounts Competitions)

Problem 4. A series of basketball games is won when one team wins three out of five games. How many combinations of wins and losses are possible if a team is to win the series? (Mathcounts Competitions)

Problem 5. A yogurt shop has four different flavors and six different toppings. If a customer wanted to get one flavor and two different toppings, how many combinations could she get? (Mathcounts Competitions)

Problem 6. In how many different ways can a panel of four on-off switches be set if no two adjacent switches may be off? (Mathcounts Competitions)

Problem 7. Six softball teams are to play each other once. How many games are needed? (Mathcounts Competitions)

Problem 8. Eight volleyball teams are to play each other twice. How many games are needed? (Mathcounts Competitions)

Problem 9. A red die and a green die are rolled. In how many of the possible outcomes is the sum of the numbers showing divisible by 2? (Mathcounts Competitions)

Problem 10. In a singles tennis tournament, each player plays every other player exactly once. There is a total of 28 games. How many players are in the tournament? (Mathcounts Competitions)

Problem 11. A flag is to be designed using 3 differently colored horizontal stripes. If 5 colors are available, how many distinct flags are possible? (Mathcounts Competitions)

Problem 12. Twenty students hold a chess tournament in which each of the students will play every other student in the group once. How many individual contests will be held? (Mathcounts Competitions)

Problem 13. How many distinct four-digit numbers are there that contain two 4's, one 5, and one 6? (Mathcounts Competitions)

Problem 14. A class of 20 students is going to form groups of 2. How many different groups are possible? (Mathcounts Competitions)

Problem 15. A teacher asks for a group of volunteers from a class of 6 students to participate in a class project. Assuming that at least one student volunteers, how many combinations of volunteers are possible? (Mathcounts Competitions)

Problem 16. How many four-digit numbers greater than 2999 can be formed such that the product of the middle two digits exceeds 5? (Mathcounts Competitions)

Problem 17. Two boys and four girls are officers of the Math Club. When the photographer takes a picture for the school yearbook, she asks the club's six officers and the faculty sponsor to sit in a row with the faculty sponsor in the middle and the two boys not next to one another. How many different seating arrangements are possible? (Mathcounts Competitions)

Problem 18. How many ways can a committee consisting of two Republicans and three Democrats be chosen from eight Republicans and six Democrats? (Mathcounts Competitions)

Problem 19. In a ten-team league, each team plays every other team exactly twice. Find the total number of games played in the league. (Mathcounts Competitions)

Problem 20. How many ways can the letters of the word DINNER be scrambled so that the first and last letters are both vowels? (Mathcounts Competitions)

Problem 21. Six teams are in the tennis-doubles playoffs. Each team plays every other team twice. What is the total number of games played? (Mathcounts Competitions)

Problem 22. How many different arrangements of the letters of the word SCIENCE have N as their first letter and S as their last? (Mathcounts Competitions)

Problem 23. How many different line segments are formed by marking seven distinct points on a line? (Mathcounts Competitions)

Problem 24. Sixteen people attended a party, and each person brought a gift for everyone else at the party. Altogether, how man gifts were brought to the party? (Mathcounts Competitions)

Problem 25. What is the total number of different committees that can be formed by selecting one or more persons from a group of six people? (Mathcounts Competitions)

Problem 26. A lock has 5 buttons numbered 1-5. The lock is opened by pushing two buttons simultaneously and then pushing one button alone. How many combinations are possible? (Mathcounts Competitions)

Problem 27. An 11-member committee makes its decisions by simple majority vote; if 6 or more of the members vote in favor of an issue, the issue is passed. In how many ways can 6 or more members vote to pass an issue? (Mathcounts Competitions)

Problem 28. Ellen has five different jobs to be done. She assigns all five jobs to her four kids. Each kid will have at least one job. How many ways can Ellen assign the jobs? (Mathcounts Competitions)

Problem 29. Two different numbers are chosen from the set {2, 3, 5, 7} and multiplied together. How many of the possible products are odd? (Mathcounts Handbooks)

Problem 30. A softball league has 8 teams. During the season, each team plays each of the other teams exactly 3 times. What is the total number of games played by all teams? (Mathcounts Handbooks)

Problem 31. A tennis team has 5 girls and 4 boys. How many mixed pairs (one boy and one girl) are possible? (Mathcounts Handbooks)

Problem 32. From a selection of six different colors, how many different flags can be made consisting of three vertical stripes if no stripes of the same color can be placed side by side? (Mathcounts Handbooks)

Problem 33. In how many ways can two dice be rolled to yield a sum divisible by 3? (Mathcounts Handbooks)

Problem 34. How many different six-digit numbers can be formed using three 5's, two 4's, and one 6? (Mathcounts Handbooks)

Problem 35. In how many different ways can 3 men and 4 women be placed into two groups of two people and one group of three people if there must be at least one man and one woman in each group? (Mathcounts Handbooks)

Problem 36. From a group of six students living in the same neighborhood, a social committee is to be appointed. Given that the committee must have at least three members, how many different committees can be formed? (Mathcounts Handbooks)

Problem 37. Six mathletes and two coaches are arranged in a line shoulder-to-shoulder for a group photo. In how many different ways can they be arranged if one coach must be at each end? (Mathcounts Handbooks)

Problem 38. Twelve students are to be divided among Mr. Mirus's and Ms. Batty's classes. No teacher is to have more than 8 students. How many different groups of students could be in Mr. Mirus's class? (2001 Mathcounts Handbook).

ANSWER KEYS:

Problem 1. 120(committees)
Problem 2. 60(ways)
Problem 3. 24(sequences)
Problem 4. 10(combinations)
Problem 5. 60
Problem 6. 8(ways)
Problem 7. 15(games)
Problem 8. 56(games)
Problem 9. 18(outcomes)
Problem 10. 8(players)
Problem 11. 60(flags)
Problem 12. 190(contests)
Problem 13. 12
Problem 14. 190(groups)
Problem 15. 63(combinations)
Problem 16. 4970(numbers)
Problem 17. 528 (arrangements) (hint: $6! - 4! \times 2 \times 2 \times 2$)
Problem 18. 560 (ways)
Problem 19. 90(games)
Problem 20. 24(ways)
Problem 21. 30(games)
Problem 22. 30(arrangements)
Problem 23. 21(segments)
Problem 24. 240(gifts)
Problem 25. 63(committees)
Problem 26. 50(combinations)
Problem 27. 1024(ways)
Problem 28. 240(ways)
Problem 29. 3
Problem 30. 84
Problem 31. 20
Problem 32. 150
Problem 33. 12
Problem 34. 60
Problem 35. 36
Problem 36. 42
Problem 37. 1440
Problem 38. 3498