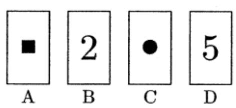
1. WARM UP PROBLEMS

Try the following problems to see how many you are able to solve. If you have trouble to solve some, read the following section to learn some skills. Then come back to working on these problems. You will find that you are in a better position to solve logic problems.

1. Squares are faster than circles, hexagons are slower than triangles, and hexagons are faster than squares. Which of these shapes is the slowest?

(A) Squares
(B) Circles
(C) Hexagons
(D) Triangles
(E) None of them

2. Four cards are constructed so that there is either a circle or a square on one side and an odd or even number on the other side. The cards are placed on a table as shown. Which cards must be turned to prove the following: Every square has an even number on the other side?



3. Classroom window was broken. The principal had four students in his office. He knew that one of them did it, and he also knew that only one of the students told the truth, but not sure which one.

Alex said: Bob did;
Bob said: Dean did;
Cam said: not me;
Dean said: Bob lied.

Who broke the window?

4. A sealed envelope contains a card with a single digit on it. Three of the following statements are true, and the other is false.
I. The digit is 1.
II. The digit is not 2.
III. The digit is 3.
IV. The digit is not 4.
Which one of the following must necessarily be correct?

(A) I is true.
(B) I is false.
(C) II is true.
(D) III is true.
(E) IV is false.

5. A centipede climbs a 40-foot tree. Each day he climbs 5 feet, and each night he slides down 3 feet. In how many days will the centipede reach the top of the tree?

(A) 19
(B) 18
(C) 17
(D) 20
(E) 21

6. Alex has 6 coins. Five of the 6 coins weigh the same and one coin is heavier. If Alex had a balance scale, what is the least number of times he could weigh coins to be sure he could determine which coin was heavier?

7. In a horse race game on a computer, Secretariat, Man-Of-War, Affirmed and Citation finished in first through fourth places (not necessarily in that order), with no ties. Man-Of-War finished second or fourth. Affirmed did not win the race. Citation or Secretariat finished third. Man-Of-War beat Secretariat. What is the name of the horse that finished fourth?

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$

8. Above are the four labeled boxes. Each box is painted a different color. There is a red box, which is next to a blue box. There is a green box, which is next to the red box and a yellow box. Which box could be painted red?

(A) 1 only
(B) 2 only
(C) 3 only
(D) 2 or 3
(E) 1 or 4

2. BASIC KNOWLEDGE REVIEW

Statements

A statement is any sentence that is either true or false, but not both.

Examples:

Boston is a city in USA.
$1 + 1 = 3$
A spider does not have six legs.

The following sentences are not statements:

Do your homework.	(a command)
How do you solve this math problem?	(a question)
SAT test is harder than ACT test.	(an opinion)
This sentence is false.	(a paradox)

Negations

The sentence "SAT math test consists of 54 problems" is a statement; the negation of this statement is "SAT math test does not consists of 54 problems".
The negation of a true statement is false, and the negation of a false statement is true.

Statement	Negation
All do	Some do not (Not all do)
Some do	None do (All do not)

Examples: Form the negation of each statement:

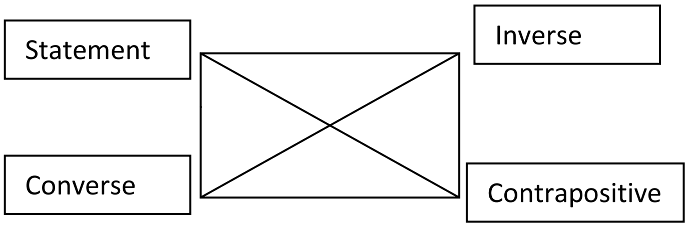
The moon is not a star.	⇒	The moon is a star.
The moon is a star.	⇒	The moon is not a star.
A spider does not have six legs.	⇒	A spider has six legs.
Some rabbits have short tails.	⇒	No rabbit has a short tail.
Some rabbits do not have short tails.	⇒	All rabbits have short tails.
No rabbit has a short tail.	⇒	Some rabbits have short tails.

Converse, Inverse, and Contrapositive

Direct statement	If p, then q.
Converse	If q, then p.
Inverse	If not p, then not q.
Contrapositive	If not q, then not p.

Direct statement	If I live in Boston, then I live in USA.
Converse	If I live in USA, then I live in Boston.
Inverse	If I do not live in Boston, then I do not live in USA.
Contrapositive	If I do not live in USA, then I do not live in Boston.

Rectangle of logical equivalent



Logically equivalent pair of statements (diagonally opposite):

A statement and its contrapositive
The inverse and converse of the same statement

Not logically equivalent pair of statements (adjacent):

A statement and its inverse
A statement and its converse
The converse and contrapositive of the same statement
The inverse and contrapositive of the same statement

Examples:

Statement:	A square is a rectangle	(true)
Converse	A rectangle is a square	(false)
Inverse	A figure that is not a square is not a rectangle	(false)
Contrapositive	A figure that is not a rectangle s is not a square	(true)

Euler Diagram

Deductive reasoning consists of three steps as follows:

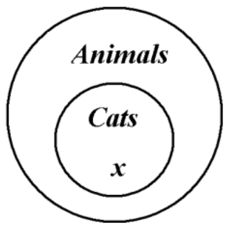
(1). Making a general statement (major premise).
(2). Making a particular statement (minor premise).
(3). Making a deduction (conclusion).

Example:

(1). The major premise is: All cats are animals
(2). The minor premise is: Jerry is a cat.
(3). The conclusion is: Jerry is an animal.

Procedures to draw the diagram:

(1) Draw a big circle to represent the first premise. This is the region for "animals".
(2) Draw a second circle to represent "all cats". Since all cats are animals, the second circle goes inside the first big circle.
(3) Put Jerry inside where it belongs. The second premise stated that Jerry is a cat. Put Jerry inside the region marked "Cats".

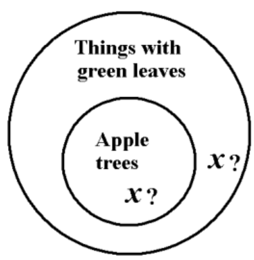


x represents Jerry

Example: Is the following argument valid? An argument is valid if that the premises are true and these premises force the conclusion to be true.

All apple trees have green leaves
That plant has green leaves.
That plant is an apple tree.

Solution: we draw the Euler Diagram. We see that "that plant" can go either inside the small circle or outside it. So the argument is not valid.



3. PROBLEM SOLVING SKILLS

(1). Find The Correct Order By Switching Positions

Example 1: Alexis, Britt, Carol, Danielle and Elizabeth are waiting in line. Alex is behind Carol but ahead of Danielle. Elizabeth is ahead of Britt, but behind Carol. Danielle is ahead of Britt. Who is first in line?

Solution: Carol is first in line.

We put them in an order like this:

A B C D E.

Since Alex is behind Carol but ahead of Danielle, we order this way:

A B C D E.

B C A D E.

Since Elizabeth is ahead of Britt, but behind Carol, we just their position like this:

B C A D E B

We check that it is true that Danielle is ahead of Britt.

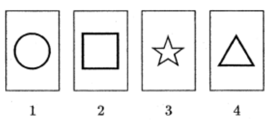
We check that it is true that Danielle is ahead of Britt. So the correct order will be like this:

C A D E B

Thus Carol is first in line.

(2). Find The Contrapositive Of The Statement

Example 2: Each card has either a circle or a star on one side and either a triangle or a square on the other side. In order to verify the statement "every card with a star on it also has a triangle on it," which numbered card(s) must be turned over?

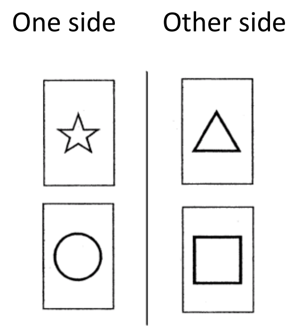


Solution: two cards (cards 2 and 3).

We introduce in this section a two-step method. This method can be used to solve any similar problems.

Step 1. We verify the statement first:

Every card with a star on it also has a triangle on it.



We must turn over every card with a star on it (card 3) to make sure it has a triangle on the other side.

Step 2. We then verify the contrapositive of the statement:

Every card without a triangle on it also does not have a star on it.

We must turn over any card without a triangle on it (in this case, card 2 with a square as shown in the figure on the left) to make sure it doesn't have a star on the other side).

(3). Find Two Statements That Are Contradicted To Each Other

Example 3: There are three boxes with different colors: red, yellow and blue. One apple is in one of the three boxes. Only one of the following statements is true, and the others are false.
I: Apple is in the red box; II Apple is not in the yellow box, and III: Apple is not in the red box.

Which box is the apple in?

Solution: The apple is in the yellow box.

First we find the two statements that are contradicted to each other. There must be a true statement between these two. Other statements left are all false.

Statement I and Statement III are two contradicted statements. We are sure that the true statement is one of these two statements, although we do know which one. So we conclude that the statement II is false. Then we know the apple is in the yellow box.

(4). Find Two Statements That Are In Agreement With Each Other

Example 4: Each of three marbles *A*, *B*, and *C*, is colored one of the three colors. One of the marbles is colored white, one is colored red, and one is colored blue. Exactly one of these statements is true:

1) *A* is red. 2) *B* is not blue. 3) *C* is not red.

What color is marble B?

Solution: *B* is while.

If 1) is true, then 3) will also be true. So these two statements are in agreement with each other. However, we know that there is only one statement is true, so it must be 2). Then we know that *C* is red. *A* is not red and *B* is not blue. Therefore, *B* is while while *A* is blue.

(5). Focus On The Step Before The Last

Example 5: A turtle crawls up a 12 foot hill after a heavy rainstorm. The turtle crawls 4 feet, but when it stops to rest, it slides back 3 feet. How many tries does the turtle make before it makes it up the hill?

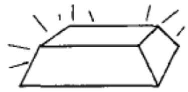
Solution: 9.

We look at where the turtle was just before the last try. Since the turtle can crawl 4 feet each time, $12 - 4 = 8$. Every try the turtle goes up 1 foot. It takes the turtle 8 tries when it reaches the 8 feet location. The turtle needs one more try to reach the top. Note when it reaches the top, there is no sliding back.

(6). Dividing Into Three Groups

When you need to weigh a number of coins with counterfeit coin, divide the coins into three groups with the number of coins in each group: m, m, m, or $m, m, m - 1$ or $m, m, m + 1$.

Example 6: A jeweler has four small bars that are supposed to be gold. He knows that one is counterfeit and the other three are genuine. The counterfeit bar has a slightly different weight than a real gold bar. Using a balance scale, what is the minimum number of weighings necessary to guarantee that the counterfeit bar will be detected?



Solution: 2.

We divide the four bars into three groups: 1, 1, and 2. We weight two bars, say, bar *A* and bar *B*, first.

Case I: If their weights are different, we remove one, say, bar *A*, and put a third bar, say bar *C*. If *B* and C are the same, and then bar *A* is the counterfeit. If bar *B* and bar C are different, bar *B* is the counterfeit (since it's weight is different from both *A* and *C*).

Case II: If their weights are the same, then we remove one, say, Bar *A*, and put a third bar, say Bar *C*. If *B* and *C* are the same, then Bar *D* is the counterfeit. If Bar *B* and Bar *C* are different, Bar *C* is the counterfeit.

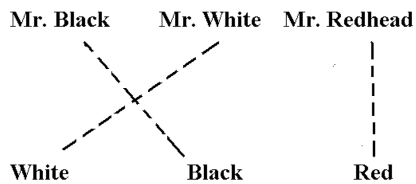
So two weighings are necessary.

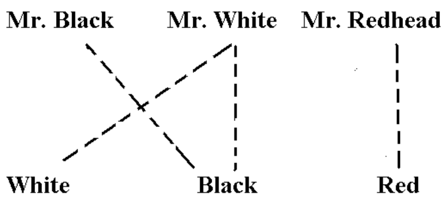
(7). Drawing Solid and Dash Lines

Example 7: Three friends – math teacher Mr. White, science teacher Mr. Black, and history teacher Mr. Redhead – met in a cafeteria. "It is interesting that one of us has white hair, another one has black hair, and the third has red hair, though no one's name gives the color of their hair" said the black-haired person. "You are right," answered White. What color is the history teacher's hair?

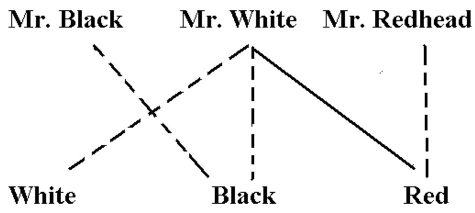
Solution: The history teacher's hair is black.
If the relationship of two things is certain (or yes), we draw a solid line between them. Otherwise, we draw a dash line.

We know that no one's name gives the color of their hair. So we draw the dash lines as shown on the right: We know that Mr. White answered the black-haired person. So he has no black hair. We draw a dash line between Mr. White and "black hair".

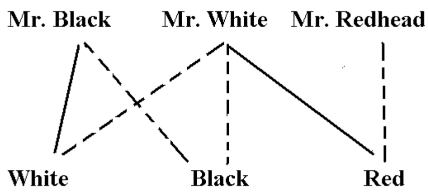




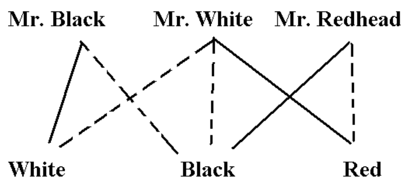
So Mr. White must have red hair. We draw a solid line to indicate that Mr. White has red hair.



Mr. Black cannot have black hair, so he must have white hair. We draw a solid line for that.



We know for sure that the history teacher's hair is black.



4. EXERCISES

1. There are 9 apparently identical balls, except that one is heavier than the other 8. What is the smallest number of balance scale weighings required to ensure identification of the "odd" ball?

(A) 9
(B) 3
(C) 4
(D) 1
(E) 2

2. A kitchen pantry has five shelves, each containing a specific kind of food. The spices are on the shelf directly below the vegetables, the fruits are above the bread, and the vegetables are 3 shelves below the cereals. Which kind of food is on the third shelf?

(A) vegetables
(B) fruits
(C) bread
(D) cereals
(E) spices

3. At Hope Middle School, Mr. Eye, Mr. Love and Mr. Problems teach science, mathematics, and history—but not necessarily in that order. The history teacher, who was an only child, has the least experience. Mr. Problems, who married Mr. Eye's sister, has more experience than the science teacher. Who teaches science?

4. Five coins look the same, but one is a counterfeit coin with a different weight than each of the four genuine coins. Using a balance scale, what is the least number of weighings needed to ensure that, in every case, the counterfeit coin is found and is shown to be heavier or lighter?

(A) 5.
(B) 4.
(C) 3.
(D) 2.
(E) 1.

5. A centipede climbs a 40-foot tree. Each day he climbs 5 feet, and each night he slides down 2 feet. In how many days will the centipede reach the top of the tree?

(A) 14
(B) 13
(C) 12
(D) 8
(E) 20

6. Adam, Ben, Charles, David and Ed were waiting in line. Adam is between Ben and Chase. Ben is between David and Adam. Ed is also between David and Adam. Ben is between David and Ed. Who is in the middle of the line?

(A) Adam
(B) Ben
(C) Charles
(D) David
(E) Ed

7. Five cards are lying on a table as shown above. Each card has a letter on one side and a whole number on the other side. Jane said, “If a vowel is on one side of any card, then an even number is on the other side.” Mary showed Jane was wrong by turning over one card. Which card did Mary turn over?

(A) 5.
(B) 4.
(C) 3.
(D) 2.
(E) 1.

8. A centipede crawl a tree 75-inches high, starting from the ground. Each day it crawls 5 inches, and each night it slides down 4 inches. When will it first reach the top of the tree?
(A) 15.
(B) 18.
(C) 19.
(D) 72.
(E) 71.

9. There are 4 cards on the table with the symbols a, b, 4, and 5 written on their visible sides. What is the smallest number of cards we need to turn over to find out whether the following statement is true: "If an even number is written on one side of a card then a vowel is written on the other side?

$\boxed{A}$ $\boxed{B}$ $\boxed{E}$ $\boxed{4}$ $\boxed{5}$ $\boxed{6}$ $\boxed{8}$

10. Each of the cards shown above has a number on one side and a letter on the other. How many of the cards must be turned over to prove the correctness of the statement: Every card with a vowel on one side has a prime number on the other side.

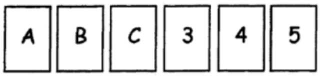
(A) 7
(B) 6
(C) 5
(D) 4
(E) 3

11. Three kids are playing pitcher, catcher and infielder. Sam is not the catcher. The infielder lives next to Sam. The catcher and John go to the same school. What position does Alex play?

12. Cookies were missing, taken by either Alex, Bob, or Charles.

Each person said:
Alex: I did not take the cookies.
Bob: Charles took the cookies.
Charles: That is true

If at least one of them lied and at least one told the truth, who took the cookies?



13. Each of the cards shown above has a number on one side and a letter on the other. How many of the cards must be turned over to prove the correctness of this statement for these cards: "If a card has a vowel on one side, then it has a prime number on the other side?"

(A) 2
(B) 2
(C) 4
(D) 5
(E) 6

14. If all alligators are ferocious creatures and some creepy crawlers are alligators, which statement(s) must be true?

I. All alligators are creepy crawlers.
II. Some ferocious creatures are creepy crawlers.
III. Some alligators are not creepy crawlers.

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) None must be true

15. A number of bacteria are placed in a container. One second later each bacterium divides into two, the next second each of the resulting bacteria divided in two again, et al. After one minute the container is full. When was the container half full?

(A) 58
(B) 59
(C) 60
(D) 120
(E) 119

Alex sometimes goes to adventure movies. Betsy never goes to comedy movies.

16. If the two statements above are true, which of the following statements must also be true?
I. Alex never goes to comedy movies.
II. Betsy sometimes goes to adventure movies.
III. Alex and Betsy never go to comedy movies together.

(A) I only
(B) II only
(C) III only
(D) I and III
(E) II and III

17. The four children in the Jones family are Alex, Bob, Cathy, and Debra. Bob is neither the youngest nor the oldest. Debra is one of the two younger children. Cathy is the oldest child. Alex is often taking care of his younger brother and sister. Who is the youngest child?

(A) Bob
(B) Debra
(C) Alex
(D) Cathy
(E) It cannot be determined from the information

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18. Sam is not a member of the math club, then from which of the following statements can it be determined whether or not Sam is in the science club?

(A) Anyone in the math club is not in the science club.

(B) No one is in both the math club and the science club.

(C) Anyone who is not in the math club is not in the science club.

(D) Everyone in the math club is in the science club.

(E) Some people who are not in the math club are not in the science club.

If a number is in list A, it is not in list B.

19. If the statement above is true, which of the following statements must also be true?

(A) If a number is not in list A, it is in list B.
(B) If a number is not in list B, it is in list A.
(C) If a number is in list B, it is not in list A.
(D) If a number is in list B, it is in list A.
(E) If a number is in list A, it is also in list B.

The first student participated in the math club. The second student did not participate in the math club. The third student participated in the reading club. The fourth student participated in the same club as the first student. The fifth student participated in the same club as the second student.

20. The Hope Middle School has three clubs: math, reading, and writing. Five students from a family each participated in one club only. The statements above are about what these five students participated. If n is the number of students who participated in the reading club, which of the following statements is true?

(A) n must be 1.
(B) n must be 2.
(C) n must be 3.
(D) n must be 1 or 2.
(E) n must be 1 or 3.

Some integers in set X are odd.

21. If the statement above is true, which of the following must also be true?

(A) If an integer is odd, it is in set X.
(B) If an integer is even, it is in set X.
(C) All integers in set X are odd.
(D) All integers in set X are even.
(E) Not all integers in set X are even.

22. If all boys in the math club are good at math. Which of the following statements must be true?

(A) No boy whose math is not good is a member of the math club.

(B) All boys whose math is good are members of the math club.

(C) All boys who are not members of the math are not good at math.

(D) Every member of the math club whose math is good is a boy.

(E) There is one boy in the math club whose math is not good.

23. At Hope High School, some members of the math club are on the science team and no members of the science team are 9th graders. Which of the following must also be true?

(A) No members of the math club are 9th graders.

(B) Some members of the math club are 9th graders.

(C) Some member of the math club are not 9th graders.

(D) More 9th graders are on the science team than are in the math club.

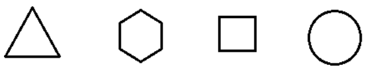
(E) More 9th graders are in the math club than are on the science team.

5. SOLUTION TO WARM UP PROBLEMS

1. Circles.
We put them in any order as shown in the figure below:

Squares are faster than circles, hexagons are slower than triangles. So we do not need to switch any order yet.

Since hexagons are faster than squares, we move the hexagon to the front. Since it is slower than triangles, we move both of them with their relative positions unchanged:



So the circles are the slowest.

2. A and D.
We verify the statement first:
Every square has an even number on the other side
We must turn over every card with a square (card A).

We then verify the contrapositive of the statement:
Every card does not have an even number on one side does not have a square on the other side. We must turn over any card with an odd number (card D) to make sure it doesn't have a square on the other side).
Cards A and D must be turned to prove the statement.

3. Cam. First we find which two statements contradiction: Bob and Dean. Then we know that both Alex and Cam did not tell the truth. Then we conclude that Cam did it. And only Dean told the truth.

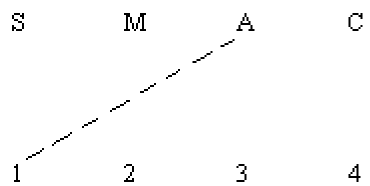
4. (C).
Method 1: We first find two statements that are in agreement with each other. In our case, II, and IV are not contradicting to each other. So we know that either I or III is the false statement. Thus II and IV must be true. From five choices, we see that (C) is correct answer.
We first find two statements that are contradicted to each other. In our case, I and III. So we know that either I or III is the false statement. Thus II and IV must be true. From the five choices, we see that (C) is correct answer.

Method 2: We first find two statements that are contradicted to each other. In our case, I and III. So we know that either I or III is the false statement. Thus II and IV must be true. From the five choices, we see that (C) is correct answer.

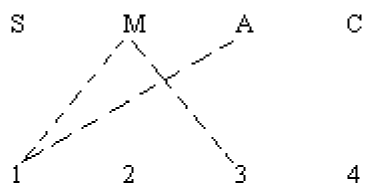
5. 19 days.
We look at where the centipede was just before the last climbing. Since the centipede can climb 5 feet each time, 40 – 5 = 35. Every time the centipede goes up 5 – 3 = 2 feet. The greatest height the centipede can go before he reaches the top is 36 feet. When the centipede reaches 36 high, he has spending 36 ÷ 2 = 18 days. The centipede needs one more day to reach the top. Note when it reaches the top, there is no sliding back.

6. 2.
Method 1: Divide the 6 coins into two groups. Each group has 3 coins. Put each group on each side of the balance scale. One side would be heavier. Taking those coins, put one on each side of the balance scale. If it balances, then the one off the scale is heavier. If it does not balance, you will know which one is heavier.
Method 2: Divide 6 coins into three groups. Each group has two coins. Taking two groups, put each group on each side of the balance scale. If it balances, then the group off the scale contains the heavier coin. The second weighing will tell which coin is heavier. If it does not balance, you take the coins from the heavier side and weight one more time to know which one is heavier.

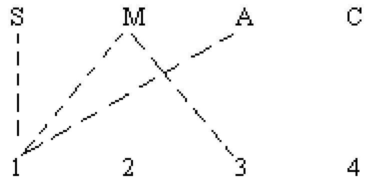
7. A.
Method 1. This method is also good for more complicated logic reasoning problems.
Since Affirmed did not win, we can draw a dash line between A and 1.



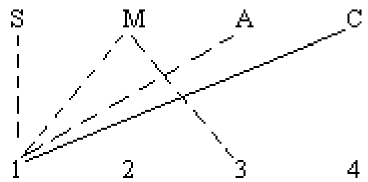
Since M finished second or fourth, then M did not finish 1 or 3. We draw a dash line between M and 1 and a dash line between M and 3.



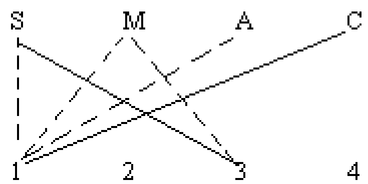
Since S was beaten by M, so S was not the first. We draw a dash line between S and 1.



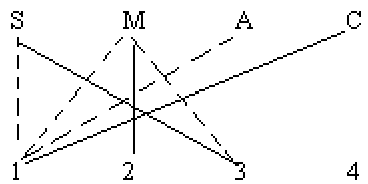
At the moment, we know that C finished first. So we draw a solid line between C and 1.



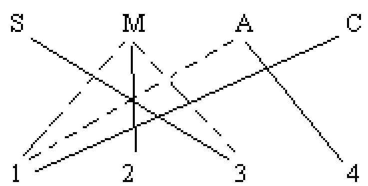
Since C or S finished third, and we know that C is not third, S must be third. We draw a solid line between S and 3.



Since M beat S. M must be second. We draw a solid line between M and 2.



The only place left for A is the fourth. So we conclude that **A finished fourth**.



Method 2:
M is 2^{nd} or 4^{th}. Since M beat S, so M can't be 4^{th} and M should be 2^{nd}. Third place has been taken by S or C, so A could not be 3^{rd}, and A did not win, so A must be 4^{th}.
8. D .

There is a red box, which is next to a blue box.
So we have two cases:

R	B
B	R

There is a green box, which is next to the red box and a yellow box.
So we also have two cases:

Y	G	R
R	G	Y

Therefore we can have two cases:

1	2	3	4
Y	G	R	B
B	R	G	Y

Therefore, either box 2 or box 3 could be painted red.

6. SOLUTION TO EXERCISES

1. 2.
We divide the 9 balls into three groups: 3, 3, and 3. We weight two groups, say, group *A* and group *B*, first.

Case I: If their weights are different, let us say, group *A* is heavier. We know that group A contains the odd ball. We divide three balls into 1, 1, and 1. We weight two of them. If these two are the same weight, then the one left is the odd ball. If these two have the different weights, the heavier one is the odd ball. So we need two weighings.

Case II: If their weights are the same, then we the group C contains the odd ball. We then follow the procedure in Case I. So we need two weighings.

Therefore two weighings are necessary.

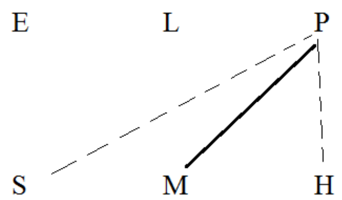
2. Bread.
We place them in any order like the following:

spices vegetables, fruits bread, cereals.

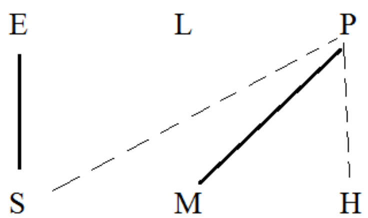
Since spices are on the shelf directly below the vegetables, we switch the position of them as follows: vegetables spices fruits bread cereals

Since the fruits are above the bread, we do not change their positions.
Since the vegetables are 3 shelves below the cereals, we move cereals to the top and switch the positions of fruits and bread as follows: cereals fruits bread vegetables spices
We are done and we know that bread is on the third shelf.

3. Mr. Eye.
We know that Mr. Problems, who married Mr. Eye's sister, has more experience than the science teacher, and the history teacher, who was an only child, has the least experience. So we are sure that Mr. Problems is neither science teacher and nor history teacher. He must be math teacher.



Mr. Eye is not the history teacher because he has a sister and the history teacher is the only child. So he must be the science teacher. We are done.



4. 3.
We divide the five coins into *A*, *B*, and *C* three groups: 2 (coins *a* and *b*), 2 (coins *c* and *d*), and 1 (coin *e*). We weight groups *A* and *B*.

Case I: If they weigh the same, coin *e* is the counterfeit coin. In this case, one weighing is needed.

Case II: If their weights are different, we remove one group, say, group *A*, and weigh the two coins *c* and *d*.

If coins c and d have the same weight, then we know that one of the coins (either *a* or *b* is the counterfeit coin. Then we weigh coins *a* and *e*. If they have the same weight, coin *b* is the counterfeit coin. . If they have the different weight, coin *a* is the counterfeit coin. We need to weigh 3 times.

If coins *c* and *d* have the different weight, then we know that one of the coins (either *c* or *d* is the counterfeit coin. Then we weigh coins *c* and *e*. If they have the same weight, coin *d* is the counterfeit coin. . If they have the different weight, coin *c* is the counterfeit coin. We also need to weigh 3 times.

So the worst case is that we need 3 weighings.

5. 13 days.

We look at where the centipede was just before the last try. Since the centipede can crawl $5 - 2 = 3$ feet each day-night, $40 = 3 \times 12 + 4$. It takes the centipede 12 tries when it reaches the 36 feet location. The centipede needs one more try to reach the top. Note when it reaches the top, there is no sliding back.

6. Ed.

We place them in any order like the following:

A B C D E

Since Adam is between Ben and Chase, we switch the position of them as follows:

B A C D E

Since Ben is between David and Adam, we move D to the front as follows:

D B A C E

Since Ed is also between David and Adam, we move D to the position below:

D B A C

E

Since Ben is between David and Ed, we know that the arrangement is as follows:

D B E A C

We are done and we know that Ed is in the middle of the line.

7. Card 3.

We verify the statement first:

If a vowel is on one side of any card, then an even number is on the other side.

We have zero card to turn over.

We then verify the contrapositive of the statement:
Every card does not have an even number on one side does not have a vowel on the other side.
We must turn over any card with a composite number (card 3 only) to make sure it doesn't have a vowel on the other side).

8. The caterpillar will be on the top of the tree at the end of the 71^{st} day.
We look at where the centipede was just before the last try. Since the centipede can crawl 5 – 4 = 1 foot each day-night, 75 = 1 × 70 + 5. It takes the centipede 70 tries when it reaches the 70 feet location. The centipede needs one more try to reach the top. Note when it reaches the top, there is no sliding back.

9. Two cards need to be turned over (Cards "4" and "b")
We verify the statement first:
"If an even number is written on one side of a card then a vowel is written on the other side".

We must turn over every card with an even number (card 4) to make sure it has a vowel on the other side.
We then verify the contrapositive of the statement:
"If a vowel is not written on the one side of a card then an even number is not written is written on the other side".

We must turn over any card not with a vowel (card b) to make sure it doesn't have an even number on the other side).
So we need to turn over 1 + 1 = 2 cards.

10. We must overturn five cards.
We verify the statement first:
Every card with a vowel on one side has a prime number on the other side.
We must turn over every card with a vowel (cards A and E) to make sure it has a prime on the other side.
We then verify the contrapositive of the statement:
Every card without a prime number on one side does not have a vowel on the other side.
We must turn over any card with a composite number (cards 4, 6, and 8) to make sure it doesn't have a vowel on the other side).

11. Alex plays catcher. Sam is not the catcher. Since John and the catcher go to the same school, John is not the catcher. Therefore, Alex is the catcher.

12. Bob took the cookies.

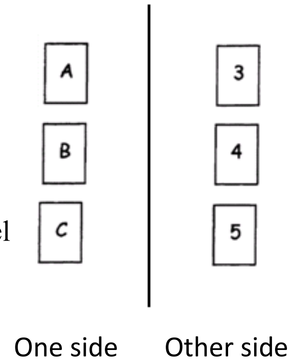
If Charles took the cookies, then all of them told the truth. If Alex took the cookies, then all of them lied. If Bob took the cookies, then Bob and Charles lied, but Alex told the truth.

13. 2 (Cards A and 4).
To verify this statement:

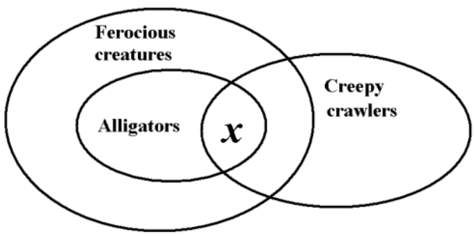
If a card has a vowel on one side, then it has a prime number on the other side
We need to turn over any card with a vowel on it. So we need to turn over card A.
We also need to test its contrapositive:

If a card does not have a prime number on one side, then it does not have a vowel on the other side"

In this case, we need to verify any card without a prime number on it (card marked 4).



14. Solution: (B)
From the conditions we can conclude that some creepy crawlers are ferocious (since some are alligators). Hence, there are some ferocious creatures that are creepy crawlers, and thus II must be true. The diagram below shows that the only conclusion that can be drawn is existence of an animal in the region with the dot. Thus, neither I nor III follows from the given conditions.



15. After 59 seconds.
We think backward and focus on the step before the last. At 59 seconds, it is half full. Then after one minute the container will be full.

16. C.
The statement "Alex sometimes goes to adventure movies" does not necessarily mean that "Alex never goes to comedy movies". So I is not necessarily correct. We can cross out (A) and (D).
For the same reason we II is not necessarily correct. We can cross out (B) and (E).
Now we can just choose the answer (C) since it is the only answer left or we can do more work as follows:

From the statement "Betsy never goes to comedy movies", we know for sure that that Alex and Betsy never go to comedy movies together. So III is correct.

17. B.
First we put them in the following order:
A B C D.
Since Cathy is the oldest, we switch her position to the left most.
C A B D.
Alex is often taking care his younger brother and sister so his position is okay.
Bob is neither the youngest nor the oldest so his position is also okay.
Therefore Debra is the youngest child

18. C.
(A) Anyone in the math club is not in the science club.
This one does not apply to Sam since he is not in the math club.
(B) No one is in both the math club and the science club.
This one does not apply to Sam since he is not in the math club.
(C) Anyone who is not in the math club is not in the science club.
This statement directly applies to Sam. Since he is not in the math club, he is not in the science club.
(D) Everyone in the math club is in the science club.
This one does not apply to Sam since he is not in the math club.
(E) Some people who are not in the math club are not in the science club.
"Some people" may or may not include Sam. So from this statement we are not able to determine whether or not Sam is in the science club.

19. C.
We only need to write out the contrapositive of the statement:
If a number is in list B, it is not in list A.
Which is (C).

20. Solution: E.
We list the possible outcomes:

1	2	3	4	5
Math	Not Math	Reading	Math	Same as 2

We see that
Case I: if the second student participated in reading club, so does the 5th student. In this case there will be 3 students who participated in the reading club.
Case II: if the second student participated in writing club, so does the 5th student. In this case there will be 1 student who participated in the reading club.
The answer is then (E).

21. (E).
If some integers in set are odd, then those odd integers are members of set that are not even. So not all integers in set are even.

22. A.
The contrapositive of the statement “all boys in the math club are good at math” is:
“If a boy’s math is not good, he is not a member of the math club”.
Which is equivalent to:
(A) No boy whose math is not good is a member of the math club.

23. (C). Some member of the math club are not 9th graders.