

Solutions to Practice Test 11

1. Solution: 2.

$$\frac{1}{2} + \frac{14}{28} + \frac{104}{208} + \frac{1004}{2008} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$$

2. Solution: 4.

$$1 * 2 = 1 + 2 \div 1 = 2$$

$$3 * 3 = 3 + 3 \div 3 = 4$$

3. Solution: 20:15:12.

$$A : B : C = \frac{5}{3}C : \frac{5}{4}C : C = \frac{5}{3} : 4 : 1 = \left(\frac{5}{3} \times 12\right) : \left(\frac{5}{4} \times 12\right) : (1 \times 12) = 20 : 15 : 12$$

4. Solution: 1.

Note the pattern of the remainders when these numbers is divided by 6:

1, 3, 3, 1, 3, 3, 1, 3, 3, Since $2011 \div 3 = 670 \text{ } r \text{ } 1$, the remainder is 1.

5. Solution: 60 days.

$$100 \times \frac{3}{3+2} = 60 \text{ days.}$$

3 days fishing every 5 days. $100 \div 5 \times 3 = 60$.

6. Solution: 50.

$$1800 \div 6 \div 6 = 50.$$

7. Solution: 879.

Method 1:

$$\overline{abc} = 100a + 10b + c.$$

$$\overline{cba} = 100c + 10b + a.$$

$$100c + 10b + a - 100a + 10b + c = 99(c - a) = 99.$$

So $c - a = 1$. We know that a , b , and c are all distinct, so the greatest value of \overline{abc} is 879.

Method 2:

$\overline{abc} + 99 = \overline{cba}$, $a = c + 9$, To make \overline{abc} the greatest, if $a = 9$, then $c = 0$, \overline{cba} is not 3-digit number, contradiction. If $a = 8$, then $c = 9$, $b = 7$, so $\overline{abc} = 879$.

8. Solution: 17.

Method 1:

$$(20 + 7 \times 2) \div 2 = 17.$$

Method 2:

Let the number of apples in bags A and B be A and B , respectively.

$$A + B = 20 \quad (1)$$

$$A - 7 = B + 7 \quad (2)$$

$$A + B = 20$$

$$(1) + (2): 2A = 34 \Rightarrow A = 17.$$

9. Solution: 34.

Group is 18, 19, 25, 26. $1 + 8 + 1 + 9 + 2 + 5 + 2 + 6 = 34$

						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	29	29
30	31					

10. Solution: \$4.

The first way: $(100 - 20) \times 0.8 = \64

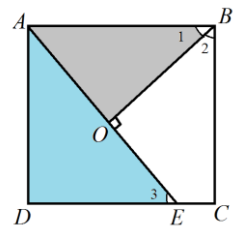
The second way: $100 \times 0.8 - 20 = \$60$.

$$64 - 60 = \$4.$$

11. Solution:

Method 1:

we label some angles as shown in the figure.



In $OECD$, $\angle 2 + \angle OEC = 180^\circ$. Since $\angle 3 + \angle OEC = 180^\circ$, $\angle 2 = \angle 3$, $\angle 1 = \angle DAE$.
 $\triangle ABO$ is similar to $\triangle EAD$.

$$\text{Thus } \frac{AB}{AE} = \frac{OB}{AD} \Rightarrow \frac{12}{AE} = \frac{9}{12} \Rightarrow AE = 16.$$

Method 2:

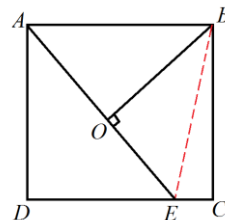
Connect BE .

$$S_{\triangle ABE} = \frac{1}{2} S_{ABCD} = \frac{1}{2} \times 12 \times 12 = 72$$

We see that OB is the height of $\triangle ABE$. Thus

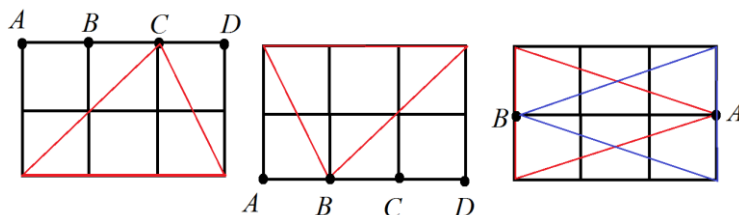
$$S_{\triangle ABE} = \frac{1}{2} AE \times OB = \frac{1}{2} AE \times 9.$$

$$\text{So } \frac{1}{2} AE \times 9 = 72 \Rightarrow AE = 16.$$



12. Solution: 10.

$6 = 1 \times 6 = 2 \times 3$. when the base is 3, we have $4 \times 2 = 8$. When the base is 2, we have $1 \times 2 = 2$. So total $8 + 2 = 10$ different triangles with an area of 3 cm^2 .



13. Solution: 9.

If one person is seated, at most we can have two empty seats going with that person.

So at most one person can reduce three seats. $27/3 = 9$. The least value of n is 9.

The following arrangement works. Each letter represents one person and each square represents one empty seat.

$\square A \square B \square C \square D \square E \square F \square G \square H \square I \square$

14. Solution: 30.

We divide the 9 numbers into three groups:

Group 1: 1, 4, 7 (the remainder is 1 when divided by 1)

Group 2: 2, 5, 8 (the remainder is 1 when divided by 2)

Group 3: 3, 6, 9 (the remainder is 1 when divided by 0)

Case 1: We take one number from each group. The sum is a multiple of 3. The number of ways to do so is $\binom{3}{1} \times \binom{3}{1} \times \binom{3}{1} = 27$ ways.

Case 2: We take three numbers from each group. The sum is a multiple of 3. The number of ways to do so is $\binom{3}{3} + \binom{3}{3} + \binom{3}{3} = 3$ ways.

The answer is $27 + 3 = 30$ ways.

15. Solution: 15.

Using the worst case scenario, we take out 4 red balls first. Next we take out 5 yellow balls. Finally we take out 5 black balls. Now we only need to take out one balls to guarantee getting 6 balls of the same color. The answer is $4 + 5 + 5 + 1 = 15$.

16. Solution: 5.

The number of candies Alex took must be a multiple of 3. $(3 + 4 + 5 + 7 + 9 + 13) = 41$.
 $41 \div 3 = 13 \text{ } r \text{ } 2$.

So the number of candy in the bag left must have a remainder of 3 when divided by 3. We see that only 5 has the remainder 2 when divided by 3 among the numbers 3, 4, 5, 7, 9, 13. So the number of candy in the bag left on the table is 5.

Method 2:

$$13 + 7 + 4 = 2 \times 3 + 9.$$

So the number of candy in the bag left on the table is 5.

17. Solution: 34.

Let a be Bob's age two years ago. $4a$ is Alex's age two years ago.

Two years from now Bob's age will be $a + 2 + 2 = a + 4$.

Two years from now Alex's age will be $4a + 2 + 2 = 4a + 4$.

$$4a + 4 = 3(a + 4) \quad \Rightarrow \quad a = 8$$

Two years from now Alex's age will be $4a + 2 + 2 = 4a + 4 = 34$.

So now Alex is $34 - 2 = 32$ years old.

18. Solution: 1.

Method 1:

$$(4 \times 30 - 110) \div (4 - 3) = 10$$

Method 2

Let c and a be the numbers of toy cars and airplanes, respectively.

$$c + a = 30 \quad (1)$$

$$4c + 3a = 110 \quad (2)$$

$$(1) \times 4 - (2): a = 10.$$

$$19. \text{ Solution: } \frac{3}{4}a - 5 + \frac{1}{2}b = \frac{1}{4}(3a + 2b) - 5 = 1.$$

20. Solution:

Method 1:

$(\$6.90 + \$22.80) = \$29.7$ can buy $(3 + 8) = 11$ pounds of apples and $2 + 9 = 11$ pounds of oranges. So the cost of 1 pound of apples and 1 pound of oranges is $\$29.7 \div 11 = \2.7 .

Method 2:

Let a and o be the costs of 1 pound of apple and 1 pound of orange, respectively.

$$3a + 2o = 6.90 \quad (1)$$

$$8a + 9o = 22.80 \quad (2)$$

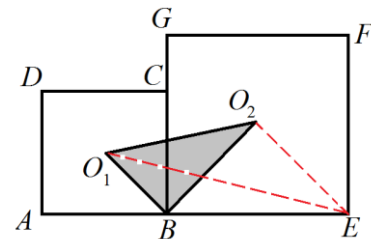
$$(1) + (2): 11(a + o) = 29.70 \quad \Rightarrow \quad a + o = 29.70 \div 11 = \$2.7.$$

21. Solution:

Method 1:

Connect O_1E , O_2E . We get $\angle O_1BA = \angle O_2EA = 45^\circ$.

So $O_1B \parallel O_2E$, and

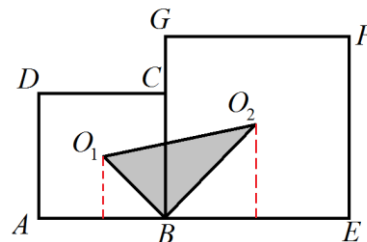


$$S_{\Delta O_2 O_1 B} = S_{\Delta O_1 B E} = \frac{1}{2} \times BE \times \frac{1}{2} \times AD = \frac{1}{2} \times 6 \times \frac{1}{2} \times 4 = 6.$$

Method 2:

The difference between the areas of one right trapezoid minus two right triangles.

$$(2 + 3) \times 5 \div 2 - 2 \times 2 \div 2 - 3 \times 3 \div 2 = 6.$$



22. Solution: 32° .

When the time is 4 pm, the angles formed by the minutes hand and the hour hand is 120° .

The minute hand moves 6° and the hour hand moves 0.5° in one minute.

After 16 minutes, the angle formed by two hands reduces $(6 - 0.5) \times 16 = 88^\circ$.

The answer is $120 - 88 = 32^\circ$.

23. Solution: 2008.

$$\frac{1}{2072} + \frac{1}{65009} = \frac{1}{8 \times 259} + \frac{1}{251 \times 259} = \frac{251 + 8}{8 \times 251 \times 259} = \frac{259}{8 \times 251 \times 259} = \frac{1}{8 \times 251} = \frac{1}{2008}.$$

So $A = 2008$.

24. Solution: 60 days.

Let B be the number of days for Bob to finish the job alone.

When Bob works for 10 days, he finishes the job: $\frac{1}{B} \times 10$

$$\left(\frac{1}{B} + \frac{1}{40}\right) \times 20 = 1 - \frac{1}{B} \times 10 \Rightarrow \frac{1}{B} \times 30 = \frac{1}{2} \Rightarrow B = 60 \text{ days.}$$

25. Solution: 7.

$$\overline{abc} + \overline{cba} = 100(a + c) + 20b + (a + c) = 101(a + c) + 20b = 888.$$

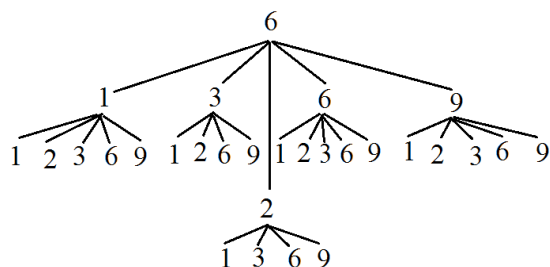
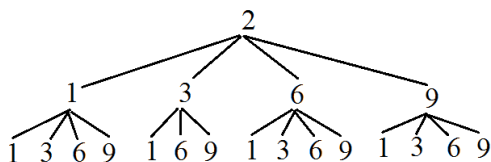
We see that the units digit of $20b$ is 0. So $a + c = 8$ and $b = 4$.

We know that both a and c are digits. Thus there are 7 solutions to $a + c = 8$.

So we have 7 such 3-digit positive integers: 147, 246, 345, 444, 543, 642, and 741.

26. Solution: 38.

To make an even integer, the units digit must be even (2 or 6). When the units digit is 2, we have 15 such numbers. When the units digit is 6, we have 23 such numbers. $15 + 23 = 38$.



27. Solution:

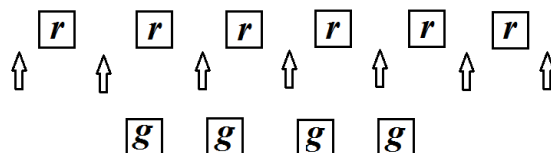
$$24300 \times (1 - 10\% - 24\% - 12\% - 36\%) = 4374.$$

28. Solution: 60° .

$$\angle BOD : \angle COD = 4 : 1, \text{ so } \angle BOC : \angle COD = 3 : 1. \angle COD = 30^\circ. \angle AOD = 60^\circ.$$

29. Solution: 35.

Alex can put 6 red balls in a row. there are seven spaces and then he can put four green balls between these red balls as shown in the figure. The number of ways is



$$\binom{7}{4} = \binom{7}{3} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35.$$

30. Solution: 5.

The number of monkeys in the end should be the factor of $56 - 1 = 55$: 1, 5, 11, or 55.

So at first, the number of monkeys could be $5 - 4 = 1$, $11 - 4 = 7$, or $55 - 4 = 52$.

Since at first a group of monkeys are dividing the pears, the number of monkeys should not be 1. 7 is a possible number for the group of monkeys first dividing the pears. Then each monkey gets $55 \div 11 = 5$ pears.

51 is not a possible number since 51 is not a factor of 56. So the answer is 5.