MATHCOUNTS

■ Mock Competition One ■

Target Round

Name ____

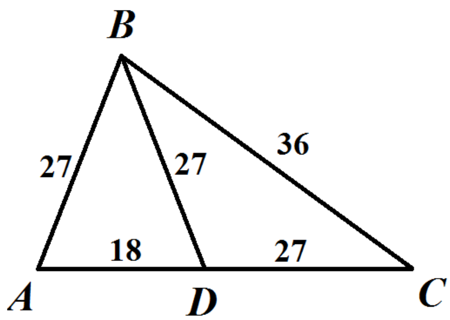
State ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of eight problems, which will be presented in pairs. Work on one pair of problems will be completed and answers will be collected before the next pair is distributed. The time limit for each pair of problems is six minutes. The first pair of problems is on the other side of this sheet. When told to do so, turn the page over and begin working. Record only final answers in the designated blanks on the problem sheet. All answers must be complete, legible, and simplified to lowest terms. This round assumes the use of calculators, and calculations may also be done on scratch paper, but no other aids are allowed. If you complete the problems before time is called, use the time remaining to check your answers.

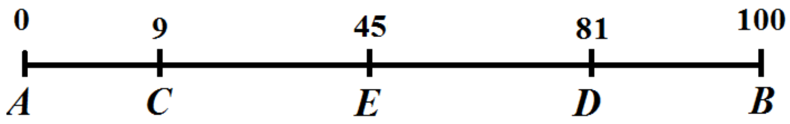
Total Correct	Scorer's Initials

1. ___________ Given triangle *ABC* as shown, one of the 5 segments, *AB*, *BC*, *CD*, *DA*, and *BD* is selected at random. A second segment is selected from the remaining four. What is the probability, expressed as a fraction, that the segments picked, in order, have lengths in the ratio 4:3 or 3:2?



2. ___________ An equilateral triangle can be formed by four smaller equilateral triangles. Find the perimeter of the equilateral triangle if the area of each smaller triangle is $25\sqrt{3}$.

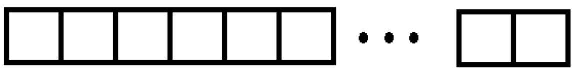
3. __________ A telephone line 100 feet long is related to a coordinate system as shown in the figure. A bird lands at *C*(9), a second bird lands at *D*(81), and a third bird lands at *E*(45). After this, birds must observe the “right of free space” of birds already on the line. An approaching bird must land at the midpoint of the longest segment available. If two segments are of equal length, the bird must select the one to the right. What is the sum of the coordinates of the points where the fifth and sixth birds must land? Express your answer as a decimal to the nearest tenth.



4. __________ A bag contains 5 blue marbles, 4 white marbles, and 3 red marbles. If three marbles are randomly selected from the bag, what is the probability that two of the three marbles selected will have the same color?

5. ___________ Find the smallest positive integer that has 2 as a remainder when divided by 3, 4, 5, 6, 7, 8, 9.

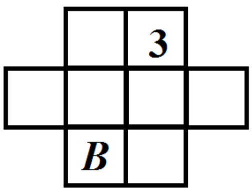
6. ___________ Any number of congruent, regular polygons (like the squares below) can be chained as shown.



Two hundred and one congruent, regular heptagons are chained in a similar manner. The resulting figure has a perimeter of 2014 centimeters. What is x, the length, in centimeters, of a side of one of the heptagons?

7. ___________ A rectangle with the length p is inscribed in a circle of radius 8 inches. Find the area inside the circle but outside the rectangle, in square inches, expressed as a function of p.

8. ___________ The eight integers, 1 through 8, are placed in the eight squares below, one integer per square, so that no two consecutive numbers are touching horizontally, vertically or diagonally. What number is placed in square *B*?



SOLUTIONS:

Problem 1: 3/10.

$36 = 2^2 \times 3^2$; $27 = 3^3$; $18 = 2 \times 3^2$.

If the ratio is 4:3, we need to have 36. This line segment goes with three segments of 27. So we have 3 ways.

If the ratio is 3:2, we need to have 27. Three line segments with the length of 27 go with the line segment 18. So we have 3 more ways.

Total number of ways is 3 + 3 = 6.

We have 5 ways to select the first line segment and 4 ways to select the second line segment.

The probability is 6/20 = 3/10.

Problem 2: Solution: 60.

The area of an equilateral triangle is given by $\frac{1}{4}a^2\sqrt{3}$, where a is the side length of the triangle.

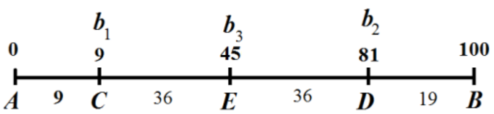
$$\frac{1}{4}a^2\sqrt{3} = 25\sqrt{3} \quad \Rightarrow \quad a^2 = 100 \quad \Rightarrow \quad a = 10.$$

The length of the sides of the larger triangle is 2 × 10 = 20.

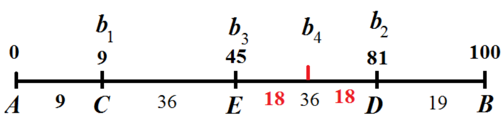
The perimeter of the larger triangle is 3 × 20 = 60.

Problem 3: Solution: 117.5.

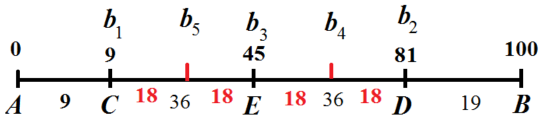
We place 3 birds on the line (b_1, b_2, and b_3) and calculate the distances for each segment:



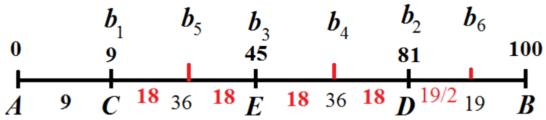
The fourth bird will land 18 feet on the right of *E*.



The fifth bird will land 18 feet on the right of *C*. The coordinate is 9 + 18 = 27.



The sixth bird will land 19/2 feet on the right of *D*.



The coordinate of the point where the sixth bird must land is 81 + 19/2 = 90.5

The answer is 27 + 90.5 = 117.5.

Problem 4: Solution: $\frac{29}{44}$.

We have three cases:

Case 1: Two blue marbles and one marble of other color:

$$P_1 = \frac{\binom{5}{2}\times\binom{7}{1}}{\binom{12}{3}} = \frac{7}{22}.$$

Case 2: Two red marbles and one marble of other color:

$$P_2 = \frac{\binom{4}{2}\times\binom{8}{1}}{\binom{12}{3}} = \frac{12}{55}.$$

Case 3: Two white marbles and one marble of other color:

$$P_3 = \frac{\binom{3}{2}\times\binom{9}{1}}{\binom{12}{3}} = \frac{27}{220}.$$

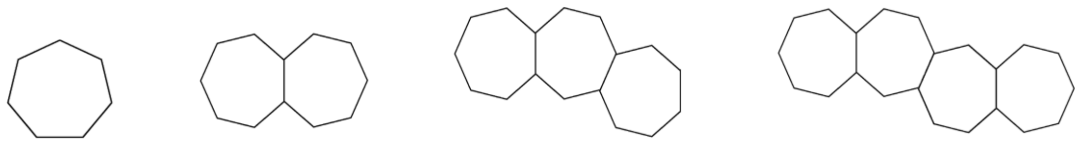
The answer is $P = P_1 + P_2 + P_3 = \frac{7}{22} + \frac{12}{55} + \frac{27}{220} = \frac{29}{44}$.

Problem 5: Solution: 2522.

The number will have a remainder 2 when divided by the lcm (3, 4, 5, 6, 7, 8, 9) = 2520. So the number is 2520 + 2 = 2522.

Problem 6. Solution: 2.

A heptagon is a polygon with seven sides and seven angles.

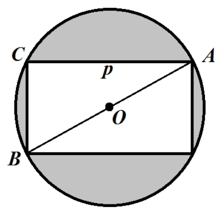


7   7 + 7 – 2   7 + 7 + 7 – 2 – 2   7 + 7 + 7 + 7 – 2 – 2– 2

The perimeter is $P = [7n - 2(n - 1)]\, x = (5n + 2)\, x$, where n is the number of heptagon.

$(5 \times 201 + 2)\, x = 2014 \quad \Rightarrow \quad x = 2.$

Problem 7: Solution: $64\pi - p\sqrt{16^2 - p^2}$.



Applying Pythagorean Theorem to right triangle ABC:

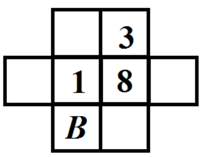
$$AC^2 + BC^2 = AB^2 \quad \Rightarrow \quad p^2 + BC^2 = 16^2 \quad \Rightarrow \quad BC = \sqrt{16^2 - p^2}\ .$$

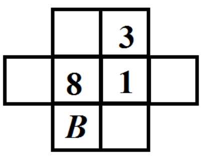
The shaded area = the area of the circle - the area of the rectangle

$$= \ \pi \times r^2 - AC \times BC = \pi \times r^2 - p\sqrt{16^2 - p^2} = 64\pi - p\sqrt{16^2 - p^2}\ .$$

Problem 8: Solution: 6.

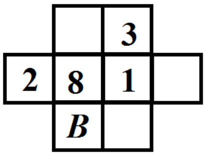
First thing we need to realize is that only digits 1 and 8 can go to the two middle squares. We have only two cases:



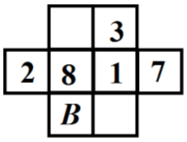


For case 1, the number 2 has no square to go. This one does not work.

For case 2, 2 can only go the left most square:

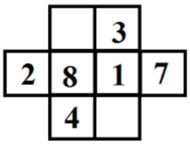


We have digits 4, 5, 6, and 7 left. 7 can only go to the left most square:

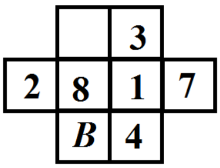


4 can be placed in *B* or the square next to *B*.

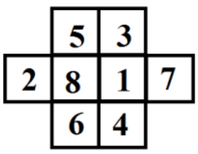
Sub case 1: If *B* is 4 as shown, we have digits 5 and 6 left. We have no way to fit them in. This sub case does not work.



Sub case 2: We place 4 as shown in the figure. We have digits 5, and 6 left.



They fit in:



The answer is then $B = 6$.