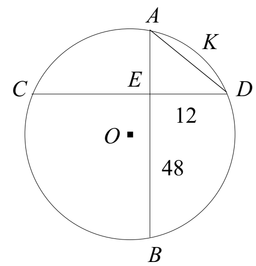
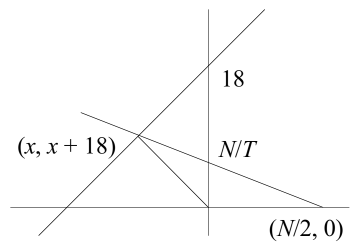
1. The 2-7-7 triangle has an area of $4\sqrt{3} < 4(2) = 8$, the 4-6-6 has an area of $8\sqrt{2} < 8(1.5) = 12$, and the 6-5-5 has an area of $\boxed{12}$. That's the largest. For a fixed perimeter, the triangle closest to an equilateral triangle will have the greatest area.

2. $T = 12$ so $K = 13$. Since $EB \cdot AE = DE \cdot CE$, then $48\sqrt{K^2 - 144} = 12 \cdot CE$. Thus, $CE = 4\sqrt{13^2 - 144} = \boxed{20}$.



3. $T = 20$ so $K = 22$. The period is $\dfrac{\pi}{\pi/K} = K = \boxed{22}$.

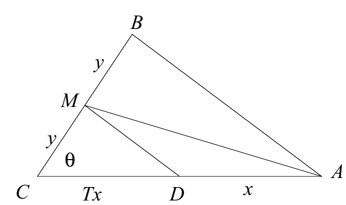
4. For a particular value of T, the y-intercept of $2x + Ty = N$ can range over the y-axis, and $2x + Ty = N$ can intersect $y = x + 18$ at any point. The point of intersection closest to the origin is just the point on $y = x + 18$ that is closest to the origin. Using $(x, x + 18)$ we need the minimum of $x^2 + (x + 18)^2$. This gives $2x^2 + 36x + 324$,



a quadratic whose minimum occurs at $x = -\dfrac{b}{2a} = -\dfrac{36}{4} = -9$. Thus, $y = -9 + 18 = \boxed{9}$. Or just find the perpendicular distance from the origin to $y = x + 18$ by finding the intersection of the line with $y = -x$.

5. $T = 9$. Since $\dfrac{\text{area } \Delta CDM}{\text{area } \Delta ABC} = \dfrac{\frac{1}{2} \cdot y \cdot Tx \cdot \sin\theta}{\frac{1}{2} \cdot 2y \cdot (Tx + x) \cdot \sin\theta} =$

$\dfrac{T}{2(T+1)} = \dfrac{9}{20}$, then $a + b = \boxed{29}$.



6. $T = 29$, so $K = 27$. The first two numbers are 18 and 81. If a three-digit number starts with 1 and the digits sum to 9 there are 9 numbers and if the digits sum to 18 there are two numbers, 189 and 198. If a three-digit number starts with 2, 3, 4, 5, 6, or 8 there are two numbers and if it starts with 7 there is just one number, 711. Through 711 there are 2 + 11 + 2 + 2 + 2 + 2 + 2 + 1 = 24 terms. In the 800's we have 801, 810, and finally 819. The 27th term in the sequence is $\boxed{819}$.

7. $T = 819$. In V_1, $2\pi r_1 = a \rightarrow r_1 = \dfrac{a}{2\pi} \rightarrow V_1 = \pi\left(\dfrac{a}{2\pi}\right)^2 b$. Similarly, $V_2 = \pi\left(\dfrac{b}{2\pi}\right)^2 a$. Thus,

$\pi \cdot \left| \pi \cdot \dfrac{a^2 b}{4\pi^2} - \pi \cdot \dfrac{b^2 a}{4\pi^2} \right| = \dfrac{ab}{4} \cdot \left| a - b \right|$. Since 819 = 3·3·7·13, pick $a = 7$ and $b = 3$, giving

$\dfrac{7 \cdot 3}{4} \cdot \left| 7 - 3 \right| = \boxed{21}$.

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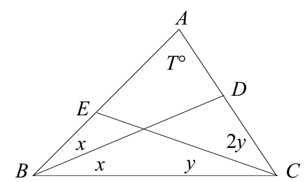
15. $12_b = b + 2 = 64 \to b = \boxed{62}$.

14. $T = 62$, so $K = 64$. Since $\log_2 x + \log_4 x + \log_8 x = \log_2 x + \log_2 x^{1/2} + \log_2 x^{1/3} = \log_2 x^{11/6} = \log_K x^n = n\log_K x$, then $\frac{11}{6} \cdot \frac{\log x}{\log 2} = \frac{n \log x}{\log K} \to n = \frac{11 \cdot \log K}{6 \cdot \log 2}$. Since $K = 64 = 2^6$, then $\log K = 6 \log 2$, making $n = \boxed{11}$.

13. Since $2x + 3y + T = 180°$, then $3x + 3y = 180° - T + x$, giving $x + y = 60° + \frac{x - T}{3}$. It is given that $x - y = \frac{T}{3}$ so adding the equations yields $2x = 60° + \frac{x}{3}$. Thus, $x = \boxed{36°}$ and T is irrelevant.



12. $T^{1/2}x^{1/2} \cdot 2^{1/3}T^{1/3}x^{2/3} \cdot 2^{1/3}T^{2/3}x^{1/2} = T^{1/2}x \cdot Tx \to 2^{2/3}T^{3/2}x^{5/3} = T^{3/2}x^2 \to 2^{2/3} = x^{1/3}$

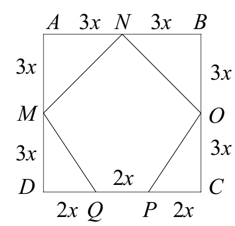
$\to x = \boxed{4}$. For the first time in ARML, T is irrelevant twice in a row.

11. $T = 4$. Since $-T^2 < Tx - T < T^2 \to 1 - T < x < 1 + T$, then if T is an integer, the largest integer value of x is T. Thus, $x = \boxed{4}$.

10. $9T = 36$. Let $AB = 6x$, then the area of $MNOPQ =$

$(6x)^2 - 2 \cdot \frac{1}{2} \cdot (3x)^2 - 2 \cdot \frac{1}{2} \cdot 2x \cdot 3x = 21x^2$. Since $x = 1$, then

$21x^2 = \boxed{21}$.



9. $T = 21$ so $K = 2$. Consider the position of the leftmost girl A and the number of positions that girl B can take. Here are the results: $\underline{K}$ $\underline{K–1}$ . . . . $\underline{3}$ $\underline{2}$ $\underline{1}$ _ _. The first seat A can sit in is the third seat from the right. Then B can only sit in the last seat. If A sits in the 4th seat from the right, then B has 2 options, if A sits in the 5th seat from the right, then B has 3 options, and so on. With A on the left, the total number of arrangements for the girls is $1 + 2 + \ldots + K = \frac{K(K+1)}{2}$. The girls can be permuted in 2! ways and the boys in K! ways so the total number of arrangements is $\frac{K(K+1)}{2} \cdot 2 \cdot K! = K \cdot (K+1)!$. Since $K = 2$, the answer is $2 \cdot 3! = \boxed{12}$. Pass $12 + 16 = 28$ to the next student. Or, note that there $K + 1$ pairs of adjacent seats and for each pair there are K! ways to arrange the boys and 2! ways to arrange the girls. From $(K + 2)!$ permutations, exclude $2!(K + 1)K!$ permutations giving $(K + 2)! - 2(K + 1)! = K(K + 1)!$

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8. Let $a = 21$ and $b = 28$. Multiply the first equation by a^2 and the second by b^2 and subtract, obtaining $\left(a^4 - b^4\right)y^2 = a^2b^2\left(a^2 - b^2\right)$, giving $y^2 = \frac{a^2b^2}{a^2 + b^2}$. Recognizing the symmetry in the system or multiplying the first by b^2, the second by a^2 and subtracting yields $x^2 = \frac{a^2b^2}{a^2 + b^2}$. Then

$x + y = \frac{2ab}{\sqrt{a^2 + b^2}} = \frac{2 \cdot 21 \cdot 28}{5 \cdot 7} = \boxed{\frac{168}{5}}$.